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EATON'S
ELEMENTS OF
ARITHMETIC.

THOMPSON, BROWN & COMPANY,
BOSTON.

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THE ELEMENTS OF WRITTEN ARITHMETIC;

COMBINING

ANALYSIS AND SYNTHESIS;

ADAPTED TO

THE BEST MODE OF INSTRUCTION

FOR BEGINNERS.

BY

JAMES S. EATON, M. A.,

INSTRUCTOR IN PHILLIPS ACADEMY, ANDOVER, AND AUTHOR OF A SERIES
OF ARITHMETICS.

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PREFACE.

DURING the few years EATON'S SERIES of ARITHMETICS has been before the Educational Public, it has been demonstrated, by their cordial reception — by their circulation, already extensive and rapidly increasing — that a deep-felt want for books of their high character has been satisfactorily met.

In the vast field at the South now being opened to educational advantages — in the extensive and rapidly growing settlements of the West, and in the cities and manufacturing districts of the older States, there is a large class of pupils whose school days are very limited. To these — next to reading and writing — a brief, practical course of Arithmetic must always form the most useful part of school training. It was to meet the wants of this class that the present work was projected by its author. Owing to an unforeseen event, its completion has devolved upon others. This has been the occasion of some delay in its publication, which however has been taken advantage of to make it as perfect as possible.

This little work then is a short, practical course of Written Arithmetic, embracing the topics actually necessary to be mastered to enable one to pursue with intelligence the ordinary business avocations of life. Special attention has been bestowed upon the FUNDAMENTAL RULES, UNITED STATES MONEY AND PERCENTAGE, and a simple but full exposition of the NEW METRIC SYSTEM is also presented.

In its preparation no labor has been spared to adapt it to the end for which it is designed. It has been submitted to many practical teachers, and thus embodies valuable suggestions from many sources. Especial credit is due Mr. J. P. Payson, Master of the Williams School, Chelsea, Mass., and it makes its way to the public through the hands of Prof. James H. Eaton, Ph. D., son of the author of Eaton's Series of Arithmetics. It is hoped that it will prove worthy to stand beside the former works of the same well-known author.

Boston, *Sept.* 30, 1867.

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ELEMENTS OF ARITHMETIC.

Article 1. A Unit is *one*, that is, *any single thing;* as, *a horse, a day, an apple, an inch.*

2. A Number is a *unit* or a *collection of units;* as, *one, two, six, three men, ten pints.*

3. Arithmetic is the *science* of numbers, and the *art* of reckoning or computation.

4. There are *six* and *only* six different operations in Arithmetic, namely, *Notation, Numeration, Addition, Subtraction, Multiplication,* and *Division.*

◆

NOTATION AND NUMERATION.

5. Notation is the art of writing or expressing numbers and their relations to each other by means of *figures and signs.*

6. Numeration is the art of reading numbers which have been written or expressed by figures.

7. There are two methods of notation in common use, namely, the *Arabic* and the *Roman.*

8. The Arabic Notation employs *ten figures* to express numbers, namely:

0,	1,	2,	3,	4,	5,	6,	7,	8,	9.
Naught,	One,	Two,	Three,	Four,	Five,	Six,	Seven,	Eight,	Nine.

Questions. **1.** What is a Unit? **2.** What is a Number? **3.** What is Arithmetic? **4.** How many operations are there in Arithmetic? What are they called? **5.** What is Notation? **6.** Numeration? **7.** How many methods of Notation in common use? What are they? **8.** How many and what figures in the Arabic Notation?

9. The first Arabic figure, 0, is called *a cipher*, *naught*, or *zero*, and when used without any other figure it stands for *nothing;* thus, 0 apples means *no* apples.

Each of the other nine figures stands for or *signifies* the same as the word written under it, and to distinguish them from 0, they are called *significant figures.*

10. To express numbers greater than nine, these figures are repeated and combined in various ways. Ten is expressed by writing the figure 1 at the left of the cipher; thus, 10. In like manner twenty, thirty, forty, etc., are expressed by placing 2, 3, 4, etc., at the left of 0; thus,

20,	30,	40,	50,	60,	70,	80,	90.
Twenty,	Thirty,	Forty,	Fifty,	Sixty,	Seventy,	Eighty,	Ninety.

11. The numbers from 10 to 20 are expressed by placing the figure 1 at the left of each significant figure; thus,

11,	12,	13,	14,	15,	16,	17,	etc.
Eleven,	Twelve,	Thirteen,	Fourteen,	Fifteen,	Sixteen,	Seventeen,	etc.

In a similar manner all the numbers, up to one hundred, may be written; thus,

21,	36,	66,	98,	etc.
Twenty-one,	Thirty-six,	Sixty-six,	Ninety-eight,	etc.

12. One hundred is expressed by placing the figure 1 at the left of *two ciphers;* thus, 100. In like manner two hundred, three hundred, etc., are written; thus,

200,	300,	600,	800,	etc.
Two hundred,	Three hundred,	Six hundred,	Eight hundred,	etc.

---

9. Names of the first figure, 0? Its value? What are the other figures called? Why? **10.** How are numbers greater than 9 expressed? Illustrate. **11.** Numbers from 10 to 20, how expressed? Other numbers to 100? One hundred, two hundred, etc., how expressed?

13. The other numbers, up to one thousand, may be expressed by putting a significant figure in the place of one or each of the ciphers in the above numbers; thus,

Two hundred and three, expressed in figures, is			203
Six hundred and eighty,	"	"	680
Nine hundred and ninety-eight,	"	"	998

14. The simple *name* of any significant figure is always the same, but the number indicated by it depends upon the place the figure occupies; for example, 6 is always *six*, and never *seven*. So in each of the following numbers, 2, 20, and 200, the left hand figure is *two*, but in the *first* it is two *units;* in the *second*, two *tens* or *twenty;* and in the *third*, two *hundreds*. Thus each significant figure has a *simple* or *name* value, and a *local* or *place* value.

15. When two or more figures are used together they are said to express *different* ORDERS *of units*. The *right hand figure* represents *simple units*, or *units of the first order;* the *second* figure represents *tens*, or *units of the second order;* the *third* represents *hundreds*, or *units of the third order;* thus, in the number 426 the 6 is simply *six*, while the 2 is two *tens* or *twenty*, and the 4 is four *hundreds;* and the number expressed by the three figures taken together is *four hundred and twenty-six*.

16. The figures of large numbers, for convenience in reading, are often separated by commas into groups or periods of three figures each, beginning at the right. The first or right-hand group contains units, tens, and hundreds,

---

13. Other numbers to one thousand? **14.** Does the *name* of a figure ever change? Does its *value* change? How many values has a figure? The *names* of those values? **15.** What is said of *orders of units*? **16.** What is said of groups or periods of figures?

and is called the *period of units;* the second period contains thousands, tens of thousands, and hundreds of thousands, and is called the *period of thousands*, etc., as in the following

NUMERATION TABLE.

Tens of Trillions, Trillions,	Hundreds of Billions, Tens of Billions, Billions,	Hundreds of Millions, Tens of Millions, Millions,	Hundreds of Thousands, Tens of Thousands, Thousands,	Hundreds, Tens, Units,
6 9,	5 4 0,	7 0 6,	4 7 6,	8 4 3,
5th period, Trillions.	4th period, Billions.	3d period, Millions.	2d period, Thousands.	1st period, Units.

17. The value of the figures in this table, expressed in words, is sixty-nine trillion, five hundred and forty billion, seven hundred and six million, four hundred and seventy-six thousand, eight hundred and forty-three.

NOTE. · The READING of a number consists of two distinct processes: First, reading the *order of the places*, beginning at the right hand; thus, units, tens, hundreds, thousands, etc., as in the *Numeration Table;* and, second, reading the *value* of the figures, beginning at the left, as in Article 17, above. To distinguish these processes, the first may be called *numerating*, and the second *reading*, the number.

18. The *value* of a figure is increased *ten fold* by removing

---

Name the periods in the Numeration Table, beginning at the right. Name the figures in each group. **17.** Read the value of the figures in the Numeration Table. NOTE. How many processes in reading a number? Describe them, and tell what they are called. **18.** How is the value of a figure affected by changing its place? Illustrate. What general law?

it *one* place toward the left; *a hundred fold* by removing it *two* places, etc., that is, ten units of the first order make one ten, ten tens make one hundred, ten hundreds make one thousand, and, in short, *ten units of any order make one unit of the next higher order.*

19. The *cipher,* when used with other figures, fills a place that would otherwise be vacant; thus, in 206 the cipher occupies the place of *tens,* because there are *no tens expressed* in the given number.

20. From the foregoing, to numerate and read a number expressed by figures, we have the following

RULE 1. *Beginning at the right, numerate, and point off the number into periods of* THREE *figures each.*

2. *Beginning at the left, read each period separately, giving the name of each period except that of units.*

EXERCISES IN NUMERATION.

21. Let the learner read the following numbers:

1.	8	11.	4,683	21.	300,006
2.	13	12.	9,000	22.	5,634,872
3.	28	13.	35,648	23.	7,402,309
4.	346	14.	40,306	24.	4,010,060
5.	704	15.	75,004	25.	2,008,004
6.	358	16.	97,400	26.	32,468,312
7.	490	17.	66,040	27.	461,084,307
8.	8,645	18.	345,284	28.	5,329,684,119
9.	3,059	19.	549,608	29.	42,382,000,000
10.	8,006	20.	203,940	30.	702,437,600,246

NOTE. The teacher should give examples similar to the above upon the blackboard or slate, sometimes inserting and sometimes omitting the commas, until the pupil can readily group, numerate,

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19. The cipher, for what used? **20.** Rule for numerating and reading a number?

and read all numbers likely to occur in his lessons or general reading. A like remark applies to all the following parts of the book. The teacher should give many original examples, varying in difficulty according to the abilities of his classes, and should encourage his pupils to make examples for themselves and for each other.

22. To write numbers, we have this

RULE 1. *Beginning at the left, write the figures belonging to the highest period.*

2. *Write the figures of each successive period in their order, filling each vacant place with a cipher.*

EXERCISES IN NOTATION.

23. Let the learner write the following numbers in figures, and read them:

1. Five units of the third order and six of the first. Ans. 506.

NOTE. A *cipher* is written in the second place, because no unit of the second order is given.

2. Three units of the fourth order, six of the second, and four of the first. Ans. 3,064.

3. Two units of the seventh order, one of the sixth, three of the third, and five of the second. Ans. 2,100,350.

4. Eight units of the fifth order, two of the third, and six of the first.

5. Six units of the eighth order, four of the sixth, two of the fourth, and five of the third.

6. Nine units of the sixth order, two of the fourth, and eight of the first.

---

23. Rule for *writing* a number. NOTE. In Notation where should 0 be written?

7. What orders of units are there in the number 3,462,895? How many units in each order?

8. What orders of units in the number 62,304,500? How many units in each order?

9. How many tens in 46? How many units beside the tens? How many units in the whole of the number?

10. In 347 how many hundreds? How many tens in the tens' place? How many units in the units' place? How many tens in the number? How many units in the number?

24. Write the following numbers in figures:

1. Two hundred and fifty-six. Ans. 256.
2. Fifty-four. Ans. 54.
3. Six thousand and nineteen. Ans. 6,019.
4. One thousand eight hundred and sixty-five.
5. Four hundred and forty.
6. Twenty-five thousand two hundred and forty-nine.
7. Two hundred and forty-five thousand six hundred and fifty-four.
8. Five million six hundred thousand eight hundred and sixteen.
9. Twenty-two million two hundred and twenty-two thousand two hundred and twenty-two.
10. Five hundred and six million forty thousand two hundred and four.
11. Four billion eight million six thousand eight hundred and ten.
12. Thirty-five trillion four hundred and six billion eight hundred and twenty million two hundred and eighteen thousand four hundred and sixty-seven.

25. The Roman Notation employs *seven capital letters* to express numbers, viz.:

I,	V,	X,	L,	C,	D,	M.
One,	**Five,**	**Ten,**	**Fifty,**	**One hundred,**	**Five hundred,**	**One thousand.**

All other numbers may be expressed by combining and repeating these letters.

26. The Roman Notation is based on the following principles:

1st. When two or more letters of equal value are united, or when one of less value *follows* one of greater, the *sum* of the values is indicated; thus, XX stands for 20, XXX for 30, LXV for 65, DC for 600, MDCCLXVIII for 1768.

2d. When a letter of less value is placed *before* one of greater, the *difference* of their values is indicated; thus, IV stands for 4, IX for 9, XL for 40, XC for 90.

3d. When a letter of less value stands *between* two of greater value, the less is to be *taken from the sum of the other two;* thus, XIV stands for 14, XIX for 19, CXL for 140.

TABLE OF ROMAN NUMERALS.

I	1	X	10	XIX	19
II	2	XI	11	XX	20
III	3	XII	12	XXI	21
IV	4	XIII	13	XXII	22
V	5	XIV	14	XXIV	24
VI	6	XV	15	XXV	25
VII	7	XVI	16	XXIX	29
VIII	8	XVII	17	XXX	30
IX	9	XVIII	18	XL	40

---

25. How many and what characters are employed in the Roman Notation? Value of each? How may other numbers be expressed?

26. What is the first principle in the Roman Notation? Second? Third?

L	50	DC	600	MDCCXLIX	1749
LX	60	DCCCC	900	MDCCCXVI	1816
XC	90	M	1000	MDCCCXLI	1841
C	100	MD	1500	MDCCCXLIX	1849
CCCC	400	MDC	1600	MDCCCLVII	1857
D	500	MDCLXV	1665	MDCCCLXVI	1866

Exercises in Roman Notation.

27. Express the following numbers by letters:

1. Nine. Ans. IX.
2. Fifteen. Ans. XV.
3. Eighteen.
4. Twenty-four.
5. Twenty-six.
6. Thirty-nine.
7. Forty.
8. Sixty.
9. One hundred and eighty-four.
10. One hundred and ninety-six.
11. One thousand six hundred and forty-six.
12. The present year, A. D. ——.

28. Besides the Arabic and Roman figures, there are various marks used to indicate certain relations between numbers and operations to be performed on them, as, for example, the *sign of equality*, $=$; the *sign of addition*, $+$; the *sign of subtraction*, $-$; etc.

These signs will be given, and their uses explained hereafter, when their aid is needed.

---

28. What characters are used in Arithmetic besides the Arabic and Roman figures? For what?

ADDITION.

29. Three apples and four apples are how many apples? *Ans. Three apples and four apples are seven apples.*

This is a question in *addition.*

30. Addition is the process of finding how many units there are in two or more numbers of the same kind taken together. The *result* of the addition is called the *sum* or *amount.*

ADDITION TABLE.

2 and 1 are 3	3 and 1 are 4	4 and 1 are 5	5 and 1 are 6
2 “ 2 “ 4	3 “ 2 “ 5	4 “ 2 “ 6	5 “ 2 “ 7
2 “ 3 “ 5	3 “ 3 “ 6	4 “ 3 “ 7	5 “ 3 “ 8
2 “ 4 “ 6	3 “ 4 “ 7	4 “ 4 “ 8	5 “ 4 “ 9
2 “ 5 “ 7	3 “ 5 “ 8	4 “ 5 “ 9	5 “ 5 “ 10
2 “ 6 “ 8	3 “ 6 “ 9	4 “ 6 “ 10	5 “ 6 “ 11
2 “ 7 “ 9	3 “ 7 “ 10	4 “ 7 “ 11	5 “ 7 “ 12
2 “ 8 “ 10	3 “ 8 “ 11	4 “ 8 “ 12	5 “ 8 “ 13
2 “ 9 “ 11	3 “ 9 “ 12	4 “ 9 “ 13	5 “ 9 “ 14
2 “ 10 “ 12	3 “ 10 “ 13	4 “ 10 “ 14	5 “ 10 “ 15
6 and 1 are 7	7 and 1 are 8	8 and 1 are 9	9 and 1 are 10
6 “ 2 “ 8	7 “ 2 “ 9	8 “ 2 “ 10	9 “ 2 “ 11
6 “ 3 “ 9	7 “ 3 “ 10	8 “ 3 “ 11	9 “ 3 “ 12
6 “ 4 “ 10	7 “ 4 “ 11	8 “ 4 “ 12	9 “ 4 “ 13
6 “ 5 “ 11	7 “ 5 “ 12	8 “ 5 “ 13	9 “ 5 “ 14
6 “ 6 “ 12	7 “ 6 “ 13	8 “ 6 “ 14	9 “ 6 “ 15
6 “ 7 “ 13	7 “ 7 “ 14	8 “ 7 “ 15	9 “ 7 “ 16
6 “ 8 “ 14	7 “ 8 “ 15	8 “ 8 “ 16	9 “ 8 “ 17
6 “ 9 “ 15	7 “ 9 “ 16	8 “ 9 “ 17	9 “ 9 “ 18
6 “ 10 “ 16	7 “ 10 “ 17	8 “ 10 “ 18	9 “ 10 “ 19

30. What is Addition? What is the *result* called?

MENTAL EXERCISES.

Ex. 1. Robert has 5 cents in one hand, and 3 cents in the other; how many cents has he in both hands? Ans. 8.

2. John bought a pencil for 6 cents, and some paper for 5 cents; how many cents did he pay for both?

3. George has 7 chickens and David has 8; how many have both? Ans 15.

4. Mary has 6 tulips and 9 roses; how many blossoms has she?

5. Daniel caught 9 fishes, Abel caught 6, and James caught 5; how many did they all catch?

6 A farmer had 6 cows in one pasture, 8 in another, and 7 in another; how many cows had he in the three pastures?

7. I paid 9 dollars for a barrel of flour, 8 dollars for a box of sugar, and 5 dollars for a cheese; how many dollars did I pay for all? Ans. 22.

8. A man bought a ton of coal for 8 dollars, a cord of wood for 6 dollars, and a stove for 9 dollars; what did he pay for all?

9. Charles has 5 marbles, Albert has 7, and Edward has 9; how many have they all?

10. A farmer has 8 sheep in one pen, 9 in another, and as many in a third pen as in both the others; how many has he in the third pen? Ans. 17.

11. A gardener raised 3 bushels of cherries, 2 bushels of currants, 5 bushels of peaches, and 8 bushels of pears; how many bushels of fruit did he raise?

12. George paid 10 cents for a writing-book, 8 cents for a pen-holder, 2 cents for pens, and 6 cents for ink; how much did he pay for all?

31. A SIGN is a mark which indicates an operation to be performed, or which is used to shorten some expression.

32. This mark, $, is often used as a sign of the word *dollar* or *dollars;* thus, $1 stands for *one dollar*, $6 stands for *six dollars.*

NOTE. It is customary to separate dollars and cents by a period; thus, $4.25 stands for *four dollars and twenty-five cents.*

33. The *sign of equality*, $=$, signifies that the quantities between which it stands are equal to each other; thus, $1 $=$ 100 cents; that is, one dollar equals one hundred cents.

34. The *sign of addition*, $+$, called *plus* or *and*, denotes that the quantities between which it stands are to be added together; thus, $3 + 2 = 5$; that is, three plus two equal five, or, three and two are five.

Ex. 12. How many are $3 + 5 + 4$? *Ans.* 12.
13. How many are $2 + 6 + 5$? $3 + 8 + 4$?
14. How many are $5 + 3 + 6$? $9 + 2 + 6$?
15. How many are $8 + 6 + 5$? $9 + 3 + 7$?
16. How many are $7 + 9 + 4$? $6 + 9 + 8$?

35. Let the pupil frequently review the following

EXERCISES IN ADDITION.

No. 1.	No. 2.	No. 3.	No. 4.	No. 5.
4 + 3	6 + 5	2 + 8	7 + 3	5 + 7
2 + 6	5 + 5	7 + 9	6 + 8	6 + 9
7 + 3	6 + 4	1 + 8	3 + 7	9 + 3
8 + 1	8 + 2	3 + 6	8 + 0	2 + 5
10 + 3	3 + 5	8 + 1	4 + 5	5 + 8
5 + 6	4 + 9	9 + 6	7 + 6	5 + 6
7 + 5	2 + 6	4 + 7	9 + 8	9 + 2

31. What is a sign? **32.** Make the sign of dollars on the black-board. How are dollars and cents separated? Give an example. Another.

33. Make the sign of equality. What does it mean? Illustrate. **34.** Make the sign of addition. What is it called? What does it mean?

No. 6.	No. 7.	No. 8.	No. 9.	No. 10.
9 + 4	1 + 4	10 + 3	2 + 1	1 + 9
6 + 7	7 + 8	8 + 10	6 + 10	0 + 8
8 + 9	8 + 5	3 + 8	5 + 11	10 + 4
6 + 2	2 + 0	2 + 7	3 + 12	6 + 11
1 + 5	5 + 4	9 + 7	9 + 0	1 + 2
2 + 2	3 + 1	1 + 3	8 + 8	10 + 7
5 + 1	2 + 4	6 + 1	9 + 9	7 + 10
No. 11.	**No. 12.**	**No. 13.**	**No. 14.**	**No. 15.**
8 + 6	3 + 9	3 + 4	4 + 8	6 + 3
10 + 5	7 + 7	7 + 2	3 + 2	7 + 12
11 + 2	6 + 6	12 + 5	3 + 3	12 + 4
6 + 12	10 + 9	10 + 6	12 + 8	11 + 11
10 + 8	12 + 6	11 + 7	10 + 11	5 + 12
9 + 1	9 + 10	9 + 12	12 + 7	11 + 9
7 + 11	8 + 12	3 + 11	11 + 8	12 + 9

WRITTEN EXERCISES.

36. To add when the numbers are large, and the amount of each column is less than ten.

Ex. 1. A farmer sold 234 bushels of corn, 423 bushels of oats, and 141 bushels of wheat; how many bushels of grain did he sell?

OPERATION.
234
423
141
———
Sum 798

Having for convenience arranged the numbers so that units stand under units, tens under tens, etc., add the units; thus, 1 and 3 are 4, and 4 are 8, and set the 8 under the column of units. Then, add the tens; thus, 4 and 2 are 6, and 3 are 9, and set the 9 under the column of tens, and so proceed till all the columns are added.

Thus we find that the entire sum is 7 hundreds, 9 tens, and 8 units, or 798 bushels the answer.

---

36. How are numbers arranged for addition? Why? Which column is added first? What is done with the sum?

In like manner add the numbers in the following examples:

Ex. 2.	3.	4.	5.	6.	7.	8.
$1.90	242	143	$26.04	324	1240	51234
2.47	126	421	12.31	23	2036	2130
3.11	211	235	41.32	241	3712	513
Ans. $7.48		799			6988	

9. Miles.	10. Bushels.	11. Men.	12. Apples.	13. Sheep.	14. Birds.	15. Days.
1310	3241	4120	4160	203	1321	3122
3247	1302	312	1306	5120	3200	2231
2131	2144	2103	2012	62	2134	2101
	6687				6655	

16. In 1850 the population of Virginia was 1,421,661, and that of Vermont was 314,120; what was the total population of Virginia and Vermont in 1850? Ans. 1,735,781.

17. In 1860 the population of Massachusetts was 1,231,065 and that of Kentucky was 1,155,713; what was the total population of Massachusetts and Kentucky in 1860?

Ans. 2,386,778.

18. A gentleman paid $135 for a horse, $243 for a chaise, and $121 for a harness; what did he pay for all?

19. Add 2346, 3120, 1201, and 2002. Ans. 8669.

20. Add $35.41, $21.24, $1.32, and $2.01. Ans. $59.98.

21. Add 43216, 20431, 14030. Ans. 77,677.

22. What is the sum of 3241 + 2312 + 1203 + 3120?

23. What is the sum of 1325 + 2312 + 1321 + 4031?

24. What is the sum of 1242 + 2123 + 1312 + 2112?

Ans. 6789.

25. What is the sum of 3124 + 1232 + 2113 + 1220?

26. What is the sum of 23102 + 52454 + 24342?

27. What is the sum of 15323 + 32354 + 41302?

37. To add when the amount of any column is ten or more.

28. A farmer raised 473 bushels of potatoes, 285 bushels of onions, 568 bushels of carrots, and 359 bushels of turnips; how many bushels of vegetables did he raise? Ans. 1685.

OPERATION.

473
285
568
359

Ans. 1685

Having arranged the numbers so that units stand under units, tens under tens, etc., as in example 1, add the numbers in the column of units; thus, 9 and 8 are 17, and 5 are 22, and 3 are 25 units, (= 2 tens and 5 units). The 5 units are set under the column of units and the 2 tens are added to the tens given in the example; thus, 2 and 5 are 7 and 6 are 13, and 8 are 21, and 7 are 28 tens (= 2 hundreds and 8 tens). The 8 tens are set under the tens, and the 2 hundreds are added to the hundreds in the example, giving 16 hundreds, or 1 thousand and 6 hundreds, which, written in their proper places, give 1685 for the answer.

38. In the same manner, add the numbers in the following *short columns*, and also add *across* the page, as suggested by the *signs*.

29.	3846 +	2843 +	63542 +	35842 +	91326 +	73241
30.	8305	3654	82735	12600	82145	38642
31.	9160	5008	23064	81264	34208	26341
	21311	11505		129706		138224
32.	3462 +	1538 +	56421 +	36245 +	35496 +	82437
33.	1354	6242	91367	24687	23549	45621
34.	1534	6215	13579	24683	35462	10820
35.	5104	3160	20013	51000	30104	28006
	11454		181380		124611	

---

37. Explain the operation in Ex. 28.

36.	4006	+	3567	+	41328	+	30000	+	5436	+	284
37.	5143		264		346		3812		46		3864
38.	5274		3806		5148		346		876		29
39.	8463		88		63405		87420		45362		389
	22886				110227				51720		

40	8716	+	501	+	432167	+	67958	+	8957351
41.	4822		9		98721		2780		2761852
42.	1920		2001		702		8765		8578127
43.	1861		92		96		83217		101
					531686				20297431

39. In solving the foregoing examples, the learner has already become familiar with all the operations in addition; but to enable him *readily to tell how to add*, we give the following

Rule. *Write the numbers in order, units under units, tens under tens, etc. Draw a line beneath, add together the figures in the units column, and if the sum be less than ten set it under the column; but if the sum be ten or more, write the units as before, and add the tens to the next column. Thus proceed till all the columns are added.*

40. Proof. *The usual mode of proof is to begin at the top and add downward. If the work is right, the two sums will be alike.*

Note 1. By this process, we *combine* the figures *differently*, and hence shall *probably* detect any mistake which may have been made in adding *upward*.

---

39. Why is a Rule for addition given? Repeat the Rule. If the amount of any column is 10 or more, where is the right-hand figure of the amount written? Why? What is done with the left-hand figure or figures? Why?

40. How is addition proved? Why not add *upward a second time?* In addition is it desirable to name the figures as we add them? Why not?

Ex. 44. In adding *upward* we say 4 and 6 are 10, and 5 are 15, and 8 are 23, etc.; but in adding *downward*, we say 8 and 5 are 13, and 6 are 19, and 4 are 23, etc.; thus obtaining the *same result*, but by different *combinations*.

ILLUSTRATION.

53468
72635
24376
27594

Sum, 178073

Proof, 178073

If we do not obtain the same result by the two methods, one operation or the other is wrong, perhaps both, and the work must be *carefully* performed again.

NOTE. In adding it is not usually desirable to name the figures that we add; thus, in Ex. 44, instead of saying 4 and 6 are 10, and 5 are 15, and 8 are 23, it is shorter and therefore better to say, 4, 10, 15, 23; and then setting down the 3, say 2, 11, 18, 21, 27, etc.

45. A grain dealer bought 3756 bushels of wheat of A, 2347 bushels of B, 1346 bushels of C, and 5468 bushels of D; how many bushels of wheat did he buy? Ans. 12917.

46. I paid $ 3465 for a farm, $15000 for a mill, $ 6795 for a lot of wool, and $ 4620 for 40 shares of railroad stock; how much did I pay for all this property? Ans. $ 29880.

47. Bought 3 city lots for $ 15345, and sold them so as to gain $ 3639; what sum did I receive for them?

Ans. $18984.

48. A man commenced trade with $ 5345, and in one year he gained $ 3462; what was he worth at the end of the year?

49. Add three hundred and twenty-five; two thousand one hundred and fifty-four; two hundred and fourteen; twenty-three thousand five hundred and forty-one; and three hundred and seventy-five. Ans. 26609.

50. What is the sum of thirty-four thousand five hun-

dred and forty-six; five million, two hundred and seventy-six thousand, four hundred and nineteen; and forty-two million, six hundred and twenty-four thousand, five hundred and eighty seven? Ans. 47,935,552.

51. England and Wales contain about 55,100 square miles; Scotland, 29,600; and Ireland, 32,000; what is the area of the British Islands?

52. The population of England in 1851 was 16,921,888; of Scotland, 2,888,742; of Wales, 1,005,721; and of Ireland, 6,515,794; what was the population of Great Britain and Ireland? Ans. 27,332,145.

53. In 1850 the population of New York was 515,547; of Philadelphia, 340,045; of Baltimore, 169,054; of Boston, 136,881; of New Orleans, 116,375; and of Cincinnati, 115,436; what was the number of inhabitants in these six cities in 1850? Ans. 1,393,338.

SUBTRACTION.

41. Three apples taken from seven apples leave how many apples? Ans. *Three apples from seven apples leave four apples.* This is a question in *Subtraction.*

42. SUBTRACTION is taking a *less number* from a *greater* number of the same kind, to find their *difference.*

The greater number is called the MINUEND; the less number, the SUBTRAHEND; and the difference, the REMAINDER.

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42. What is Subtraction? What the Minuend? Subtrahend? Remainder?

SUBTRACTION TABLE.

1 from 2 leaves 1	2 from 3 leaves 1	3 from 4 leaves 1
1 " 3 " 2	2 " 4 " 2	3 " 5 " 2
1 " 4 " 3	2 " 5 " 3	3 " 6 " 3
1 " 5 " 4	2 " 6 " 4	3 " 7 " 4
1 " 6 " 5	2 " 7 " 5	3 " 8 " 5
1 " 7 " 6	2 " 8 " 6	3 " 9 " 6
1 " 8 " 7	2 " 9 " 7	3 " 10 " 7
1 " 9 " 8	2 " 10 " 8	3 " 11 " 8
1 " 10 " 9	2 " 11 " 9	3 " 12 " 9
1 " 11 " 10	2 " 12 " 10	3 " 13 " 10
4 from 5 leaves 1	5 from 6 leaves 1	6 from 7 leaves 1
4 " 6 " 2	5 " 7 " 2	6 " 8 " 2
4 " 7 " 3	5 " 8 " 3	6 " 9 " 3
4 " 8 " 4	5 " 9 " 4	6 " 10 " 4
4 " 9 " 5	5 " 10 " 5	6 " 11 " 5
4 " 10 " 6	5 " 11 " 6	6 " 12 " 6
4 " 11 " 7	5 " 12 " 7	6 " 13 " 7
4 " 12 " 8	5 " 13 " 8	6 " 14 " 8
4 " 13 " 9	5 " 14 " 9	6 " 15 " 9
4 " 14 " 10	5 " 15 " 10	6 " 16 " 10
7 from 8 leaves 1	8 from 9 leaves 1	9 from 10 leaves 1
7 " 9 " 2	8 " 10 " 2	9 " 11 " 2
7 " 10 " 3	8 " 11 " 3	9 " 12 " 3
7 " 11 " 4	8 " 12 " 4	9 " 13 " 4
7 " 12 " 5	8 " 13 " 5	9 " 14 " 5
7 " 13 " 6	8 " 14 " 6	9 " 15 " 6
7 " 14 " 7	8 " 15 " 7	9 " 16 " 7
7 " 15 " 8	8 " 16 " 8	9 " 17 " 8
7 " 16 " 9	8 " 17 " 9	9 " 18 " 9
7 " 17 " 10	8 " 18 " 10	9 " 19 " 10

MENTAL EXERCISES.

Ex. 1. Joseph has 8 marbles in his right hand, and 5 in his left hand; how many more marbles has he in his right hand than in his left? Ans. 3.

2. Thomas paid 10 cents for a melon, and 4 cents for an orange; how much more did the melon cost than the orange?

3. Daniel paid $12 for a colt and $5 for a lamb; how much less did the lamb cost than the colt?

4. A boy having 15 peaches gave away 8 of them; how many had he remaining?

5. A man owing $17 paid $9; how much did he then owe?

6. Bought goods for $9 and sold them for $13; how much did I gain? Ans. $4.

7. Sold goods for $15, which was $6 more than they cost me; what did I pay for them?

8. William is 18 years old and George is 9 years younger; how old is George?

9. John had 17 cents and spent 9 of them; how many cents had he then?

10. A tailor had 15 yards of cloth, from which he sold 9 yards; how many yards remained? Ans. 6.

11. Samuel is 16 years old and David is 9; how much older is Samuel than David?

12. Isaac had 12 marbles, but has lost 7 of them; how many marbles has he now?

43. The *sign of subtraction*, —, called *minus* or *less*, signifies that the number after it is to be taken from the number before it; thus, $7 - 4 = 3$; that is, seven minus four, or seven less four, equals three.

Ex. 13. How many are $9 - 5$? Ans. 4.

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43. Make the sign of Subtraction on the black-board. What is it called? What does it mean? Illustrate.

14. How many are 8 — 6? 12 — 3? 10 — 7?
15. How many are 12 — 5? 9 — 6? 11 — 5?
16. How many are 16 — 7? 15 — 9? 13 — 8?
17. How many are 17 — 6? 12 — 8? 18 — 9?
18. How many are 18 — 7? 16 — 9? 14 — 9?

44. Let the pupil frequently review the following

Exercises in Subtraction.

No. 1.	No. 2.	No. 3.	No. 4.	No. 5.
6 — 2	9 — 5	7 — 4	8 — 6	13 — 6
8 — 5	6 — 3	9 — 6	7 — 7	6 — 4
3 — 1	10 — 4	12 — 5	7 — 0	8 — 3
9 — 7	7 — 5	8 — 7	9 — 3	10 — 5
7 — 3	3 — 3	4 — 1	2 — 2	7 — 1
5 — 4	3 — 0	5 — 3	6 — 5	11 — 3
4 — 2	7 — 2	7 — 6	15 — 9	12 — 4
2 — 1	8 — 4	9 — 8	12 — 8	15 — 10
No. 6.	**No. 7.**	**No. 8.**	**No. 9.**	**No. 10.**
9 — 4	10 — 6	12 — 9	15 — 7	14 — 8
8 — 2	12 — 3	14 — 6	17 — 9	16 — 6
12 — 6	16 — 4	11 — 7	14 — 5	10 — 9
10 — 7	11 — 5	4 — 3	10 — 8	13 - 4
12 — 7	18 — 2	9 — 2	9 — 1	15 — 5
14 — 2	14 — 7	11 — 6	10 — 0	18 — 7
16 — 7	8 — 1	12 — 10	10 — 10	17 — 8
15 — 8	11 — 4	15 — 6	18 — 8	14 — 4
No. 11.	**No. 12.**	**No. 13.**	**No. 14.**	**No. 15.**
17 — 6	15 — 11	18 — 6	17 — 11	16 — 12
16 — 9	17 — 10	16 — 10	18 — 5	18 — 4
14 — 10	14 — 9	14 — 12	16 — 8	17 — 5
18 — 9	11 — 8	12 — 1	13 — 9	14 — 3
12 — 11	16 — 5	15 — 4	15 — 3	17 — 7
15 — 12	10 — 3	13 — 8	18 — 16	16 — 12
13 — 7	13 — 5	18 — 12	16 — 2	13 — 11
18 — 10	18 — 11	15 — 13	14 — 11	11 — 9

WRITTEN EXERCISES.

45. To subtract when no figure in the subtrahend is greater than the figure above it.

Ex. 1. From 837 take 523. Ans. 314.

OPERATION.
Minuend, 837
Subtrahend, 523
Remainder, 314

Having written the less number under the greater, units under units, tens under tens, etc., we say 3 from 7 leaves 4, 2 from 3 leaves 1, and 5 from 8 leaves 3; therefore the remainder is 314.

In like manner solve the following examples:

	Ex. 2.	3.	4.	5.	6.
From	$ 53.68	$ 736.45	38697	386495	836942
Take	$ 21.43	$ 325.13	13543	243345	314241
Ans.	$ 32.25	$ 411.32		143150	

	7.	8.	9.	10.	11.
	Hours.	Men.	Women.	Children.	Horses.
From	9368	65439	63548	390642	897436
Take	3215	25316	21410	180321	135223
		40123		210321	

12. By the census of 1860, there were 326072 inhabitants in New Hampshire, and 628276 in Maine; how much less was the population of New Hampshire than of Maine?

13. By the census of 1860, the population of Mississippi was 791396, and that of the United States Territories was 220143; how many more people were there in Mississippi than in the Territories? Ans. 571253.

14. A farmer bought a farm for $ 3465, and sold it for $ 4689; how much did he gain? Ans. $ 1224.

15. How many are 29 less 16? 876 less 346?

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45. How are numbers arranged for Subtraction? Why? Which figure is subtracted first? Where is the Remainder written?

16. How many are 89 less 74? 963 less 241?
17. How many are 836 — 215? 8360 — 6320?
18. How many are 869 — 349? 9386 — 2150?

46. To subtract when any figure in the subtrahend is greater than the figure above it.

19. From 863 take 249. Ans. 614.

OPERATION.

Minuend,	863
Subtrahend,	249
Remainder,	614

Two methods for explaining this operation are in common use.

1st. As we cannot take 9 units from 3 units, *one* of the 6 *tens* is put with the 3 units, making 13 units, and then, 9 units from 13 units leave 4 units, which is set under the units. Now, as *one* of the 6 *tens* has been used, only 5 tens remain in the minuend, and 4 tens from 5 tens leave 1 ten, and, finally, 2 hundreds from 8 hundreds leave 6 hundreds; therefore the entire remainder is 614.

2d. We may add 10 units (equal to 1 *ten*) to the three units, making 13 units. From this sum we subtract the 9 units. In subtracting the next column, instead of *taking away* 1 of the 6 tens in the minuend, we may *add* 1 ten to the 4 tens in the subtrahend, and then take the *sum* (5 tens) from the 6 tens, and the result is 1 ten as by the former process.

The *second* mode depends on the principle, that *if two numbers are equally increased, the difference between them remains unchanged.* Now, in solving Ex. 19 by the second method, we add 10 *units* to the *minuend,* and 1 *ten (the same as* 10 *units)* to the *subtrahend,* and therefore find the *same remainder* as by the first method.

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46. How many methods of subtracting when a figure of the subtrahend is greater than the figure over it? Explain the first method. Explain the second. The second depends on what principle? *Is the same number added to minuend and subtrahend? How?*

47. In the same manner solve the following examples, taking each lower number from the one over it in each example; also, subtract in the manner indicated by the *signs*.

20.	86326 —	43710	53684 —	36146	74663 —	53462
	34613 —	23620	21392 —	19324	43158 —	25319
	51713		32292	16822		28143
21.	58327 —	36418	66888 —	43682	83624 —	34261
	43618 —	18294	51364 —	35176	52842 —	21638
		18124		8506		12623
22.	73926 —	53614	83654 —	46839	93654 —	9342
	61498 —	39182	8263 —	3642	8432 —	584
		14432		43197		8758
23.	99594 —	74660	46832 —	39481	73162 —	68243
	81940 —	50706	36481 —	22814	61928 —	34821
		23954			11234	

48. The pupil having become familiar with the modes of subtracting, we aid him by giving the following:

RULE 1. *Write the less number under the greater, units under units, tens under tens, etc., and draw a line beneath.*

2. *Beginning at the right hand, take each figure in the subtrahend from the figure above it, and set the remainder under the line.*

3. *If any figure in the subtrahend is greater than the figure above it, add* TEN *to the upper figure and take the lower figure from the* SUM; *set down the remainder, and considering the next figure in the minuend* ONE LESS, *or the next figure in the subtrahend* ONE GREATER, *proceed as before.*

---

48. What is the Rule for Subtraction?

49. PROOF. *Add the subtrahend and the remainder together, and the sum should be the minuend.*

NOTE 1. This proof rests on the self-evident truth, that *the whole of a thing is equal to the sum of all its parts;* thus, the *minuend* is separated into the *two parts, subtrahend and remainder; hence* the sum of those parts *must be* the *minuend.*

ILLUSTRATION.

Minuend,	8264
Subtrahend,	3692
Remainder,	4572
Proof,	8264

Ex. 24. As the *sum* of the subtrahend and remainder is the minuend, the work is supposed to be right.

	25. Inches.	26. Men.	27. Gallons.	28. Apples.
From	8365	636554	966482	835670
Take	4879	482732	381779	482984
Rem.,	3486		584703	
Proof,	8365			

	(7)	(9)	(13)
From	8	0	3
Take	2	6	7
Ans.	5	3	6

29. Here, we cannot take 7 from 3, nor can we borrow from the tens' place, as that place is occupied by 0; but we can borrow *one* of the 8 *hundreds,* and separate the one hundred into 9 *tens* and 10 *units;* then, putting the 9 tens in the place of tens, and adding the 10 units to the 3 units in the minuend, we can subtract 7 from 13, 6 from 9, and 2 from 7.

NOTE 2. This process will probably be more readily *understood* by the young learner than the second method given in the rule, though the latter, for convenience, is usually adopted.

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49. What the proof? On what principle does the proof rest? Illustrate. Explain Ex. 29. Which mode of subtracting is more readily understood? Which more convenient?

	30.	31.	32.	33.	34.
		Days.	Sheep.	Miles.	Bushels.
From	$5304	6403	5030	9084	8005
Take	$2457	3846	2684	7692	3689
	$2847		2346		4316

35. Washington was born in 1732, and died in 1799; at what age did he die? Ans. 67.

36. How many years have passed since the discovery of America in 1492?

37. Jamestown, in Virginia, was settled in 1607; how many years from that date was the Declaration of Independence in 1776? Ans. 169.

38. Queen Victoria was born in 1819; how old was she in 1865? Ans. 46.

39. A merchant bought goods for $3846, and sold the same for $5050; what was his gain? Ans. $1204.

40. A merchant paid $8004 for goods, and sold the same for $6846; what was his loss?

41. How many years from the discovery of America by Columbus, in 1492, to the settlement of Plymouth by the Puritans, in 1620? Ans. 128.

42. In 1864 a man died at the age of 87 years; in what year was he born?

43. The sum of two numbers is 80304, and the greater number is 54836; what is the less? Ans. 25468.

44. The less of two numbers is 34685, and their sum is 90304; what is the greater? Ans. 55619.

45. The difference between two numbers is 3684, and the greater number is 8002; what is the less?

46. From one thousand eight hundred and sixty-five take one thousand four hundred and ninety-two.

47. From two million, three hundred and sixty-one thousand, four hundred and seventeen, take one million, five hundred and forty-six thousand, two hundred and eighty-nine. Ans. 815128.

48. Suppose the distance from the earth to the sun is 94879956 miles, and that from the earth to the moon is 240000 miles; how much farther is the sun than the moon from the earth?

49. The population of the United States was 31,443,790 in 1860, and 23,191,876 in 1850; what was the increase in ten years? Ans. 8,251,914.

50. Suppose the outstanding public debt of the United States to be $ 2,800,000,000, and that $ 125,375,287 now in the treasury be applied to its payment, what would then be their indebtedness?

EXAMPLES IN ADDITION AND SUBTRACTION.

1. From the sum of 94 and 86, take 117. Ans. 63.

2. From the sum of the three numbers, 629, 493, and 896, take the sum of 968 and 563. Ans. 487.

3. I owe three notes, whose sum is $ 3895; one of these notes is for $ 1348, another for $ 863; for how much is the third? Ans. $ 1684.

4. A farmer having 1275 acres of land, sold 318 acres at one time, 227 at another, and 175 at another; how many acres has he remaining?

5. If a man's income is $1865 a year, and he pays $200 for rent, $ 468 for food, $ 278 for clothing, and $ 712 for other expenses, how much will he save in the year? Ans. $ 207.

7. How many are 876 + 392 + 847 — 963?

8. How many are 986 + 389 + 549 — 846?

9. Two men start from the same place and travel in the same direction, one goes 125 miles, the other 876, how far apart are they? How far if they had travelled in opposite directions?

10. From 7,000,000 subtract 8901 + 101.

Ans. 6990998.

11. A man purchased a farm for $ 6890, and having paid $ 575 for an additional piece of land, he sells the whole for $ 7500; does he gain or lose, and how much?

12. In a Union school there are four departments; in the first there are 125 scholars, in the second 379, in the third 437, and in the fourth 487; how many scholars does it contain? If 692 are boys, how many are girls?

First Ans. 1428. Second Ans. 736.

13. A general started out on a campaign with three regiments of soldiers, the first numbered 1025 men, the second 975, the third 875; after a battle he finds but 2575 in all reported fit for duty; how many men has he lost?

14. How many are 687 + 594 + 369 — 918?

15. In 1850, the population of New York was 515547; that of Philadelphia, 340045; of Baltimore, 169054; and of Boston, 136881. At the same time, the population of London was about 2363241; what was the difference between the population of London and the aggregate population of the four cities named in the United States? Ans. 1201714.

16. A merchant bought some flour for $ 347, some rye for $ 236, and some oats for $ 563; he sold the whole for $ 1275. Did he gain or lose? How much?

17. Mr. Jones gives $ 2376.43 to his four sons, as follows: to Daniel, $ 534.68; to James, $ 354.68; to Thomas, $ 486.39; and the rest to David. What does David receive?

Ans. $ 1000.68.

MULTIPLICATION.

50. In 1 bushel there are 32 quarts; how many quarts are there in 8 bushels?

1st Method, BY ADDITION.	2d Method, BY MULTIPLICATION.
3 2	3 2
3 2	8
3 2	——
3 2	Product, 2 5 6
3 2	
3 2	
3 2	
3 2	
——	
Sum, 2 5 6	

This example may be solved by *addition*, as by the 1st method; but as there will evidently be 8 times as many quarts in 8 bushels as there are in 1 bushel, it may be more briefly solved as by the 2d method; thus, 8 times 2 units are 16 units, = 1 ten and 6 units; write the 6 units in the place of units, and then say 8 times 3 tens are 24 tens, which, increased by the 1 ten previously obtained, make 25 tens, = 2 hundreds and 5 tens, and when these are written in their proper places we have 256 quarts for the true result. This, when solved by the 2d method, is a question in *multiplication.*

51. MULTIPLICATION is a short method of *adding equal numbers;* or, it is a short method of finding how many units there are in any number of times a given number.

The *number repeated* is called the MULTIPLICAND; the *number showing how many times* the multiplicand is taken is the MULTIPLIER; the *sum*, or *result* of the multiplication, is the PRODUCT. The *Multiplicand* and *Multiplier* are called FACTORS.

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50. Explain the two methods of solving the above example. Which is best?

51. What is Multiplication? Another definition. What is the Multiplicand? Multiplier? Product? Factors? **52.** Repeat the Multiplication Table.

52. The pupil, before advancing further, should learn the following

MULTIPLICATION TABLE.

Once	Twice	Three times	Four times	Five times	Six times
1 is 1	1 are 2	1 are 3	1 are 4	1 are 5	1 are 6
2 " 2	2 " 4	2 " 6	2 " 8	2 " 10	2 " 12
3 " 3	3 " 6	3 " 9	3 " 12	3 " 15	3 " 18
4 " 4	4 " 8	4 " 12	4 " 16	4 " 20	4 " 24
5 " 5	5 " 10	5 " 15	5 " 20	5 " 25	5 " 30
6 " 6	6 " 12	6 " 18	6 " 24	6 " 30	6 " 36
7 " 7	7 " 14	7 " 21	7 " 28	7 " 35	7 " 42
8 " 8	8 " 16	8 " 24	8 " 32	8 " 40	8 " 48
9 " 9	9 " 18	9 " 27	9 " 36	9 " 45	9 " 54
10 " 10	10 " 20	10 " 30	10 " 40	10 " 50	10 " 60
11 " 11	11 " 22	11 " 33	11 " 44	11 " 55	11 " 66
12 " 12	12 " 24	12 " 36	12 " 48	12 " 60	12 " 72

Seven times	Eight times	Nine times	Ten times	Eleven times	Twelve times
1 are 7	1 are 8	1 are 9	1 are 10	1 are 11	1 are 12
2 " 14	2 " 16	2 " 18	2 " 20	2 " 22	2 " 24
3 " 21	3 " 24	3 " 27	3 " 30	3 " 33	3 " 36
4 " 28	4 " 32	4 " 36	4 " 40	4 " 44	4 " 48
5 " 35	5 " 40	5 " 45	5 " 50	5 " 55	5 " 60
6 " 42	6 " 48	6 " 54	6 " 60	6 " 66	6 " 72
7 " 49	7 " 56	7 " 63	7 " 70	7 " 77	7 " 84
8 " 56	8 " 64	8 " 72	8 " 80	8 " 88	8 " 96
9 " 63	9 " 72	9 " 81	9 " 90	9 " 99	9 " 108
10 " 70	10 " 80	10 " 90	10 " 100	10 " 110	10 " 120
11 " 77	11 " 88	11 " 99	11 " 110	11 " 121	11 " 132
12 " 84	12 " 96	12 " 108	12 " 120	12 " 132	12 " 144

MENTAL EXERCISES.

1. How many leaves will 7 sheets of paper make if each sheet is folded in 8 leaves? Ans. 56.

2. In how many days can I do as much work as 9 men can do in 5 days?

3. One peck contains 8 quarts; how many quarts are there in 3 pecks?

4. If 9 yards of cloth are required to make 1 garment, how many yards are required to make 8 such garments?

5. How many men can do as much work in 1 day as 6 men can do in 9 days? Ans. 54.

6. If you solve 9 examples each hour, how many examples will you solve in 9 hours?

7. When flour is worth $ 10 per barrel, how much must be paid for 7 barrels?

8. If 12 inches make 1 foot, how many inches are there in 3 feet? Ans. 36.

9. In 1 year there are 12 months; how many months are there in 2 years? 4 years? 3 years? 7 years? 5 years? 8 years?

10. If I deposit $10 a month in a savings bank, how much shall I deposit in 5 months? In 4 months? In 8 months? In 9 months?

53. The *sign of multiplication*, ×, signifies that the two numbers between which it stands are to be multiplied together; thus, $6 \times 5 = 30$; that is, six multiplied by five equals thirty, or six times five are thirty.

Ex 11. How many are 7×4? Ans. 28.

---

53. Make the sign of multiplication on the black-board. What does it signify?

54. Review until familiar the following

EXERCISES IN MULTIPLICATION.

No. 1.	No. 2.	No. 3.	No. 4.	No. 5.
3 × 5	4 × 6	8 × 4	8 × 7	6 × 3
6 × 4	7 × 3	7 × 9	6 × 6	7 × 5
5 × 8	6 × 8	5 × 6	5 × 0	9 × 5
4 × 7	5 × 4	4 × 9	5 × 1	8 × 6
7 × 8	8 × 3	6 × 5	3 × 7	7 × 4
9 × 2	9 × 4	7 × 6	4 × 8	6 × 7
2 × 8	6 × 9	5 × 7	7 × 2	2 × 9
1 × 6	3 × 8	8 × 8	9 × 3	3 × 6

No. 6.	No. 7.	No. 8.	No. 9.	No. 10.
7 × 7	10 × 3	1 × 8	12 × 8	11 × 7
8 × 5	8 × 2	7 × 10	7 × 11	12 × 6
10 × 2	6 × 1	12 × 5	9 × 8	9 × 10
3 × 9	7 × 10	8 × 10	4 × 10	10 × 6
5 × 5	6 × 2	7 × 12	8 × 1	4 × 12
8 × 9	12 × 3	12 × 4	12 × 2	4 × 4
4 × 5	4 × 11	8 × 11	9 × 9	6 × 11
9 × 6	5 × 12	9 × 7	11 × 3	8 × 12

No. 11.	No. 12.	No. 13.	No. 14.	No. 15.
12 × 7	11 × 6	12 × 11	10 × 10	11 × 9
10 × 8	10 × 7	11 × 12	11 × 11	5 × 3
6 × 12	5 × 11	3 × 11	9 × 12	11 × 2
6 × 0	4 × 3	5 × 9	3 × 10	10 × 11
6 × 1	5 × 10	9 × 11	2 × 12	10 × 12
10 × 9	12 × 1	3 × 12	11 × 8	12 × 10
12 × 9	10 × 5	11 × 5	1 × 1	4 × 2
11 × 4	12 × 10	10 × 4	2 × 11	12 × 12

WRITTEN EXERCISES.

55. To multiply by a single figure.

Ex. 1. Multiply 879 by 6. Ans. 5274.

FIRST OPERATION.

```
  879
    6
  ---
   54 units,
  42  tens,
 48   hundreds.
 ----
 5274
```

Since 879 is to be multiplied by 6, each order of its units must be multiplied by 6; hence the product must consist of 54 units, 42 tens, and 48 hundreds; and, therefore, the product is

Instead of writing the products of the units, tens, etc., separately and adding the several partial products, it is more convenient, and, therefore, customary to multiply as in the

SECOND OPERATION.

```
 879
   6
----
5274
```

Here, by the same plan as in Article 50, we say, 6 times 9 are 54; set down the 4, and then say, 6 times 7 are 42, and 5 are 47; set down the 7, and then say, 6 times 8 are 48 and 4 are 52; set the 2 and 5 in their proper places, and the entire product, is the same as before.

56. From the above we have the following

RULE. *Write the multiplier under the multiplicand, and draw a line beneath; multiply the units of the multiplicand, set the units of the product under the multiplier, and add the tens, if any, to the product of the tens, and so proceed till the example is solved.*

---

55. Explain the First Solution of Example one. Explain the Second Solution. Are the two methods alike in principle? Which is the more convenient in practice? **56.** Which figure of the multiplicand is multiplied first? Where are the units of the product written? What is done with the tens? Repeat the Rule.

	Ex. 2.	3.	4.
Multiplicand,	$5.36	642	954
Multiplier,	7	4	8
Product,	$37.52	2568	

5.	6.	7.
9843	3645	634508
2	9	3
19686		

8.	9.	10.
439684	389642	823465
5	7	6
2198420		4940790

11.	12.	13.
854839	$9086.47	856732
9	9	4
	81778.23	

14. Multiply 63842 by 5. Ans. 319210.
15. Multiply 30687 by 7.
16. Multiply 86054 by 6. Ans. 516324.
17. Multiply 72309 by 4.
18. Multiply 59480 by 8. Ans. 475840.
19. Multiply 40069 by 9.
20. Multiply 562347 by 2.
21. Multiply 385462 by 3.
22. How many are 3584 × 7? Ans. 25088.
23. How many are 8639 × 4?
24. If 1 farm is worth $6375, what are 6 farms worth, at the same price?

25. In 1 square foot there are 144 square inches; how many square inches are there in 9 square feet?

26. In 1 solid foot there are 1728 solid inches; how many solid inches are there in 8 solid feet? Ans. 13824.

57. To multiply by a number expressed by two or more figures.

27. In 1 bushel there are 32 quarts; how many quarts are there in 48 bushels?

OPERATION.

Multiplicand,	32
Multiplier,	48
Partial Products,	256
	128
True Product,	1536

First multiply by 8, as though 8 were the only figure in the multiplier; then multiply by 4, and set the first figure of this product in the place of *tens*; for multiplying by the 4 *tens* is the same as multiplying by 40, and 40 times 2 units are 80 *units* = 8 *tens*; that is, the product of *units* by *tens* is *tens*. Having multiplied by each figure in the multiplier, the *sum* of the *partial products* is the *true product*.

58. Similar reasoning applies however many figures there may be in the multiplier. Hence,

RULE 1. *Set the multiplier under the multiplicand, and draw a line beneath.*

2. *Beginning at the right hand of the multiplicand, multiply the multiplicand by each figure of the multiplier, setting the first figure of each partial product directly under the figure of the multiplier which produces it.*

3. *The* SUM *of the partial products will be the true product.*

---

57. Explain the solution of Example 30. Where is the first figure of each partial product written? Why? **58.** Repeat the Rule.

59. **Proof.** *Multiply the multiplier by the multiplicand, and, if correct, the result will be like the first product.*

Note. This proof rests on the principle, that the *order* of the factors is immaterial; 3×4=4×3; 5×2×7=2×7×5=2×5×7, etc.

Ex. 28. Multiply 569 by 478.

	OPERATION.		PROOF.
Multiplicand,	569		478
Multiplier,	478		569
	4552		4302
	3983		2868
	2276		2390
Product,	271982	=	271982

60. In the same manner solve the following examples, multiplying each upper number by the one under it in each example; also multiply in the manner indicated by the *signs*.

29.	8 × 9	24 × 17	634 × 376	4362 × 3264
	6 × 7	21 × 14	428 × 245	2468 × 1357
30.	7 × 6	36 × 24	568 × 492	5486 × 3698
	9 × 8	27 × 32	634 × 346	2534 × 4368
31.	9 × 9	46 × 54	648 × 396	8682 × 3842
	8 × 6	35 × 43	827 × 423	6342 × 4362
32.	9 × 5	88 × 77	986 × 684	9999 × 6843
	8 × 7	64 × 72	379 × 793	4682 × 7953

33. If a steamboat sails 12 miles per hour, how far will she sail in 24 hours; that is in 1 day?

34. If a steamboat sails 288 miles per day, how far will she sail in 28 days?

---

59. Proof? Principle?

35. What is the value of 37 acres of land, at $ 13.36 per acre? Ans. $494.32.

36. How long will it take 1 man to do as much work as 24 men can do in 75 days? Ans. 1800 days.

37. How far will a horse travel in 27 days, if he travels 37 miles per day?

38. How many yards of cloth in 33 pieces, if each piece contains 54 yards? Ans. 1782.

39. Multiply two hundred and fifteen thousand eight hundred and forty-seven by six hundred and fifty-nine.

Ans. 142,243,173.

40. What is the cost of building 243 miles of railroad, at $ 48,750 per mile?

41. If a garrison of soldiers eat 5876 pounds of bread per day, how much will they eat in 365 days?

61. Ciphers between the significant figures of the multiplier may be neglected, taking care to set the first figure of each partial product directly under the figure of the multiplier which gives that product.

Ex. 42. Multiply 7543 by 2005.

OPERATION.

```
            7543
            2005
           -----
           37715
        15086
        --------
Product, 15123715
```

This is only carrying out the principle (in addition) of setting units under units, tens under tens, etc. The 2 of the multiplier is 2000, and 2000 times 3 is 6000, therefore the 6 of the partial product should be written in the place of thousands; that is, directly under the 2 of the multiplier.

43. Multiply 3642 by 3008. Ans. 10955136.

---

61. What may be done with ciphers between the significant figures of the multiplier? What care is required? Principle?

44. What cost 507 miles of railroad at $ 3648 per mile? Ans. $ 1,849,536.

45. How many lemons in 806 boxes, if each box contains 309 lemons?

46. How many pounds of cotton in 3004 bales, each bale containing 537 pounds? Ans. 1,613,148.

CONTRACTIONS.

62. The rules already given are sufficient for all examples that can arise in multiplication, but there are various devices for shortening the process in particular cases.

63. To multiply by a composite number.

A COMPOSITE NUMBER is the *product* of two or more numbers; 15 is a composite number, whose factors are 3 and 5; 12 is a composite number, whose factors are 2 and 6, or 3 and 4, or 2, 2, and 3. It will be observed that a composite number *may have* several sets of factors.

47. How many dollars have 35 men, if each man has $ 43?

OPERATION.

35 = 5 × 7.

Multiplicand,	$ 43
1st Factor of Multiplier,	5
	$ 215
2d Factor of Multiplier,	7
Product,	$ 1505

The 35 men may be separated into 7 groups of 5 men each. Now 1 group of 5 men will have 5 times $ 43, = $ 215, and if 1 group has $ 215, evidently 7 groups will have 7 times $ 215 = $ 1505, Ans. That is, 7 times 5 times a number are 35 times that number.

---

62. What is said of the rules already given for Multiplication? What of shorter modes?

63. What is a composite number? May a composite number have more than *one set* of factors? Rule for multiplying by a composite number?

48. Multiply 367 by 168. Ans. 61656.

	FIRST OPERATION.		SECOND OPERATION.
	168 = 8 × 7 × 3.		168 = 4 × 7 × 6.
Multiplicand,	367		367
1st Factor of Multiplier,	8		4
	2936		1468
2d Factor of Multiplier,	7		7
	20552		10276
3d Factor of Multiplier,	3		6
Product,	61656	=	61656

Several other sets of factors of 168 may be used, and give the same product. Every similar example may be solved in like manner. Hence,

Rule. *Multiply the multiplicand by one of the factors of the multiplier, and that product by another factor, and so on until all the factors in the set have been taken; the last product will be the true product.*

49. Multiply $ 8.37 by 36. Ans. $ 301.32.
50. Multiply $ 659 by 56.
51. Multiply 737 by 72.
52. Multiply 967 by 96.
53. Multiply 8356 by 81.
54. Multiply 6753 by 49.
55. Multiply 7045 by 54.

64. To multiply by 10, 100, 1000, or 1 with any number of ciphers annexed.

Rule. *Annex as many ciphers to the multiplicand as there are ciphers in the multiplier, and the number so formed will be the product.*

---

64. How is a number multiplied by 10? By 100? Why?

Note. The reason of the rule is obvious. Annexing a cipher removes each figure in the multiplicand one place toward the left, and thus its value is increased ten fold (Art. 18).

56. Multiply 74 by 10. Ans. 740.
57. Multiply 357 by 1000. Ans. 357000.
58. What is 8769 × 100?
59. What is 3568 × 10,000?
60. What is 9806 × 100,000?

65. To multiply by 20, 50, 500, 25000, or any number, with ciphers at the right:

Rule. *Multiply by the significant figures, and to the product annex as many ciphers as there are ciphers at the right of the significant figures of the multiplier.*

61. Multiply 756 by 30. Ans. 22680.

OPERATION.

$$\begin{array}{r} 756 \\ 30 \\ \hline 22680 \end{array}$$

This is upon the principle of Art. 63. The factors of 30 are 3 and 10. Having multiplied by 3, the product is multiplied by 10 by annexing 0 (Art. 64).

62. Multiply 743 by 3500, using factors.

OPERATION.

$$\begin{array}{r} 743 \\ 7 \\ \hline 5201 \\ 500 \\ \hline \end{array}$$

The factors of 3500 are 7, 5, and 100, therefore multiply first by 7, then by 5, then annex two ciphers.

Product, 2600500.

63. Multiply 5386 by 42000. Ans. 226212000.
64. Multiply 6539 by 240000. Ans. 1569360000.
65. Multiply 9743 by 630000.

---

65. How is a number multiplied by 20? Why?

66. To multiply when there are ciphers at the right of both multiplicand and multiplier :

Rule. *Multiply the significant figures of the multiplicand by those of the multiplier, and then annex as many ciphers to the product as there are ciphers at the right of both factors.*

66. Multiply 8000 by 900. Ans. 7200000.

OPERATION.

8 0 0 0
9 0 0

Ans. 7 2 0 0 0 0 0

The factors of 8000 are 8 and 1000, and those of 900 are 9 and 100. Now, as it is immaterial in what order the factors are taken (Art. 59, Note), first multiply 8 by 9, then multiply this product by 1000 (Art. 64), and this product by 100.

67. Multiply 730000 by 2900.

OPERATION.

7 3 0 0 0 0
2 9 0 0

6 5 7
1 4 6

Product, 2 1 1 7 0 0 0 0 0 0.

68. Multiply 37000 by 29000. Ans. 1073000000.

69. 730000 by 47000. Ans. 34310000000.

70. 17000 by 79000000.

71. 4500 by 720000. Ans. 3240000000.

67. To multiply by 9, 99, or any number of 9's.

Rule. *Annex as many 0's to the multiplicand as there are 9's in the multiplier, and from the number so formed subtract the multiplicand ; the remainder will be the product sought.*

---

66. Rule when there are ciphers at the right of both factors? Reason?
67. Rule for multiplying by 9, 99, 999, etc.? Reason?

72. Multiply 234 by 99.

OPERATION.

23400 = 100 times the multiplicand.
234 = 1 time the multiplicand.
23166 = 99 times the multiplicand, Ans.

73. Multiply 5379 by 999. Ans. 5373621.

74. Multiply 638 by 9999. Ans. 6379362.

75. Multiply 739 by 99. By 999.

Examples in the Foregoing Principles.

1. A merchant bought 156 barrels of flour at $ 9 per barrel, and 75 barrels at $12 per barrel. He also sold 987 bushels of wheat at $ 2 per bushel; how much more did he pay for the flour than he received for the wheat? Ans. $ 330.

2. Two men start from the same place, and travel in the same direction, one at the rate of 48 miles, and the other 36 miles per day; how far apart are they at the end of 17 days? Ans. 204 miles.

3. Had the men named in Ex. 2 travelled in *opposite* directions, how far apart would they have been in 31 days? Ans. 2604 miles.

4. A farmer killed 2 oxen weighing 975 pounds each, 3 cows weighing 462 pounds each, and 5 swine weighing 456 pounds each; how much more beef than pork had he? Ans. 1056 pounds.

5. The President of the United States receives a salary of $ 25000 a year; what will he save in a year of 365 days, if his expenses are $ 60 a day? Ans. $ 3100.

6. A man having a journey of 287 miles to perform in 5 days, travels 62 miles a day for 4 days; how far must he go on the fifth day?

7. Bought a herd of 25 cows, paying as many dollars for each cow as there were cows in the herd. Paid $ 500 in money, and gave my note for the balance; what was the amount for which the note was given? Ans. $ 125.

8. Bought 13 cows at $ 42 each, and 21 pair of oxen at $ 87 a pair; what did I pay for all? Ans. $ 2373.

9. Sold 3 farms; for the first I received $ 2345, for the second, $ 364 less than for the first, and for the third, twice as much as for the other 2; how much did I receive for the 3 farms? Ans. $ 12978.

10. The factors of one number are 31 and 43, and of another 29 and 17; what is the difference of the two numbers?

11. A teacher receives $ 1200 a year, and pays $ 364 a year for board, $ 96 for clothes, $ 75 for books, and $ 356 for other expenses; how much will he save in 5 years?

12. A manufacturer receives $ 37950 in one year for the products of a certain factory. For materials he pays out $ 15675, for labor $ 10369, for repairs of machinery $ 2006; how much profit remains to him? Ans. $ 9900.

13. If the above manufacturer, after paying out of his income $ 5 on every hundred dollars for United States tax, $ 3 on every hundred for other taxes, expends $ 4875 for the support of his family, how much remains? Ans. $ 4233.

14. A drover bought 280 head of cattle for an average cost of $ 75 per head, 10 horses for $ 210 each; after deducting the expenses of transporting them to the market he found he had made $ 15 per head on the cattle, $ 50 apiece on the horses, what was the amount of his profits?

15. A steamboat makes 300 trips in one season; she carries an average of 225 passengers each trip, and 75 tons of merchandize. If the average receipts are $ 2 for each passenger and $ 1 for each ton of freight, how much money does she receive? Ans. $ 157500.

16. A country merchant went to the city to purchase goods, carrying with him $3000. He bought 20 barrels of flour, at $12 per barrel, 275 gallons of molasses, at $1 per gallon, a box of sugar for $178, two pieces of broadcloth at $56 a piece; other dry goods to the amount of $525, and other groceries to the amount of $118, and a variety of small goods to the amount of $375. After paying for these how much money had he left?

DIVISION.

68. How many peaches, at 2 cents each, can I buy for 6 cents? Ans. *As many as* 2 *cents is contained times in* 6 *cents; therefore I can buy* 3 *peaches, because* 2 *cents is contained* 3 *times in* 6 *cents.*

If 12 apples are divided equally among 3 boys, how many apples will each boy have? Ans. *Each boy will have* 4 *apples, because if* 12 *apples are divided into* 3 *equal parts each part is* 4 *apples.* These are questions in *Division.*

69. DIVISION is the process of finding how many times one number is contained in another; or, DIVISION is the process of separating one number into as many equal parts as there are units in another number.

The *number to be divided* is called the DIVIDEND; the *number by which to divide* is the DIVISOR; the *number of times* the dividend contains the divisor is the QUOTIENT; and, if the dividend does not contain the divisor an *exact number of times,* the *part of the dividend that is left* is the REMAINDER.

NOTE. The remainder, *being a part of the dividend,* is always of the *same kind* as the dividend.

---

68. Explain the Examples in Art. 68. **69.** What is Division? Another definition? What is the Dividend? Divisor? Quotient? Remainder? Of what kind is the remainder? Why?

DIVISION TABLE.

1 in 1	Once.	2 in 2	Once.	3 in 3	Once.
1 in 2	Twice.	2 in 4	Twice.	3 in 6	Twice.
1 in 3	3 times.	2 in 6	3 times.	3 in 9	3 times.
1 in 4	4 “	2 in 8	4 “	3 in 12	4 “
1 in 5	5 “	2 in 10	5 “	3 in 15	5 “
1 in 6	6 “	2 in 12	6 “	3 in 18	6 “
1 in 7	7 “	2 in 14	7 “	3 in 21	7 “
1 in 8	8 “	2 in 16	8 “	3 in 24	8 “
1 in 9	9 “	2 in 18	9 “	3 in 27	9 “
4 in 4	Once.	5 in 5	Once.	6 in 6	Once.
4 in 8	Twice.	5 in 10	Twice.	6 in 12	Twice.
4 in 12	3 times.	5 in 15	3 times.	6 in 18	3 times
4 in 16	4 “	5 in 20	4 “	6 in 24	4 “
4 in 20	5 “	5 in 25	5 “	6 in 30	5 “
4 in 24	6 “	5 in 30	6 “	6 in 36	6 “
4 in 28	7 “	5 in 35	7 “	6 in 42	7 “
4 in 32	8 “	5 in 40	8 “	6 in 48	8 “
4 in 36	9 “	5 in 45	9 “	6 in 54	9 “
7 in 7	Once.	8 in 8	Once.	9 in 9	Once.
7 in 14	Twice.	8 in 16	Twice.	9 in 18	Twice.
7 in 21	3 times.	8 in 24	3 times.	9 in 27	3 times.
7 in 28	4 “	8 in 32	4 “	9 in 36	4 “
7 in 35	5 “	8 in 40	5 “	9 in 45	5 “
7 in 42	6 “	8 in 48	6 “	9 in 54	6 “
7 in 49	7 “	8 in 56	7 “	9 in 63	7 “
7 in 56	8 “	8 in 64	8 “	9 in 72	8 “
7 in 63	9 “	8 in 72	9 “	9 in 81	9 “

MENTAL EXERCISES.

Ex. 1. How many oranges, at 5 cents apiece, can be bought for 15 cents? Ans. *As many as* 5 *cents is contained times in* 15 *cents, namely,* 3.

2. At 5 cents an ounce, how many ounces of cloves can be bought for 30 cents? Ans. 6.

3. At $ 6 a cord, how many cords of wood can I buy for $ 24.

4. At $ 8 a ton, how many tons of coal can I buy for $ 24? For $ 40? For $ 56? For $ 32? Last Ans. 4.

5. In how many weeks, at $ 9 a week, will a man earn $ 27? $ 54? $ 36? $ 63?

6. At $ 9 a barrel, how many barrels of flour can I buy for $ 45? For $ 81? For $ 63?

7. In how many hours will a horse travel 36 miles if he travels 6 miles per hour? If 9 miles? If 4 miles?

8. When blueberries cost 10 cents a quart, how many quarts can be bought for 40 cents? For 70 cents? 90 cents?

9. How many sheep, at $ 11 apiece, can I buy for $ 55? For $ 44? For $ 66? Last Ans. 6.

10. How many pounds of coffee, at 12 cents a pound, can I buy for 36 cents? For 48 cents? For 72 cents?

11. Two men, 72 miles apart, approach each other at the rate of 9 miles per hour; in how many hours will they meet?

12. I divided 15 cents equally among 5 boys; how many cents did each boy receive? Ans. *If* 15 *cents are divided into* 5 *equal parts, each part is* 3 *cents, therefore each boy received* 3 *cents.*

13. A farmer sold 5 sheep for $ 45; what was their average price?

14. If 9 men can cut 54 cords of wood in a week, how many cords can 1 man cut in the same time?

15. A pile of 48 barrels of apples will exactly fill 8 equal bins; how many barrels will each bin hold?

16. I divided 55 cents equally among 11 boys; how many cents did each receive?

17. A dairy woman has 84 pounds of butter which she wishes to divide equally among her 12 customers; how many pounds can she furnish each?

18, If a workman earns $ 48 in one month of 4 weeks, how much does he earn in one week?

19. How much will the above workman earn in one day?

20. How many bushels of wheat, at $ 3 per bushel, will it take to pay for 15 bushels of rye, at $ 2 per bushel?

21. If 7 suits of clothes can be made from 54 yards of cloth, how many yards does it take for one suit?

22. If the cloth for one of the above suits costs $ 21, how much is that per yard?

70. The *sign of division*, ÷, indicates that the number before it is to be divided by the number after it; thus, 8 ÷ 2 = 4; that is, 8 divided by 2 equals 4; or, 2 in 8, 4 times.

Ex. 23. How many are 10 ÷ 2? Ans. 2 *in* 10, 5 *times.*
24. How many are 9 ÷ 3? 12 ÷ 4? 10 ÷ 5?
25. How many are 15 ÷ 5? 20 ÷ 4? 36 ÷ 9?
26. How many are 16 ÷ 8? 48 ÷ 6? 63 ÷ 7?
27. How many are 49 ÷ 7? 63 ÷ 9? 64 ÷ 8?
28. How many are 72 ÷ 9? 72 ÷ 8? 72 ÷ 6?
29. How many are 84 ÷ 7? 45 ÷ 9? 56 ÷ 7?
30. How many are 81 ÷ 9? 54 ÷ 6? 48 ÷ 8?

---

70. Make the sign of Division on the blackboard. What does it indicate?

71. Review frequently the following

EXERCISES IN DIVISION.

No. 1.	No. 2.	No. 3.	No. 4.	No. 5.
8 ÷ 4	16 ÷ 4	6 ÷ 3	35 ÷ 5	30 ÷ 6
18 ÷ 6	27 ÷ 3	12 ÷ 4	42 ÷ 6	16 ÷ 8
15 ÷ 3	45 ÷ 9	24 ÷ 6	72 ÷ 8	36 ÷ 9
49 ÷ 7	16 ÷ 2	42 ÷ 7	24 ÷ 4	14 ÷ 7
30 ÷ 5	32 ÷ 8	40 ÷ 5	9 ÷ 9	20 ÷ 2
8 ÷ 1	35 ÷ 7	48 ÷ 8	18 ÷ 3	28 ÷ 4
24 ÷ 8	36 ÷ 6	54 ÷ 9	14 ÷ 2	15 ÷ 5
18 ÷ 2	25 ÷ 5	18 ÷ 2	56 ÷ 7	21 ÷ 3

No. 6.	No. 7.	No. 8.	No. 9.	No. 10.
30 ÷ 10	44 ÷ 11	56 ÷ 8	48 ÷ 12	7 ÷ 7
63 ÷ 9	72 ÷ 9	36 ÷ 12	84 ÷ 7	40 ÷ 10
45 ÷ 5	54 ÷ 9	70 ÷ 7	12 ÷ 3	60 ÷ 12
36 ÷ 4	40 ÷ 8	12 ÷ 6	77 ÷ 11	81 ÷ 9
48 ÷ 6	4 ÷ 4	20 ÷ 4	6 ÷ 6	40 ÷ 4
63 ÷ 7	30 ÷ 3	27 ÷ 9	60 ÷ 4	55 ÷ 11
64 ÷ 8	21 ÷ 7	50 ÷ 10	20 ÷ 5	80 ÷ 8
12 ÷ 2	55 ÷ 5	60 ÷ 5	8 ÷ 8	50 ÷ 5

No. 11.	No. 12.	No. 13.	No. 14.	No. 15.
72 ÷ 12	88 ÷ 11	60 ÷ 10	22 ÷ 11	132 ÷ 12
80 ÷ 10	70 ÷ 10	96 ÷ 12	108 ÷ 12	50 ÷ 5
77 ÷ 7	99 ÷ 9	90 ÷ 9	100 ÷ 10	66 ÷ 6
60 ÷ 6	96 ÷ 8	88 ÷ 8	48 ÷ 4	110 ÷ 10
36 ÷ 3	33 ÷ 3	24 ÷ 12	28 ÷ 7	132 ÷ 11
22 ÷ 2	24 ÷ 2	99 ÷ 11	120 ÷ 12	110 ÷ 11
12 ÷ 1	84 ÷ 12	90 ÷ 10	121 ÷ 11	20 ÷ 10
66 ÷ 11	33 ÷ 11	108 ÷ 9	120 ÷ 10	144 ÷ 12

72. Division is indicated not only by the sign ÷, given in Art. 70, but also by the *colon*, thus, 8 : 2; also, by writing the divisor *before* the dividend, with a curved line between them, thus, 2) 8; also, by writing the divisor *under* the dividend, with a line between them, thus, $\frac{8}{2}$; each of which expressions means that 8 is to be divided by 2.

73. The last mode of indicating division, sometimes called the *fractional sign*, gives the following compact

DIVISION TABLE.

$\frac{1}{1} = 1$	$\frac{2}{2} = 1$	$\frac{3}{3} = 1$	$\frac{4}{4} = 1$	$\frac{5}{5} = 1$	$\frac{6}{6} = 1$
$\frac{2}{1} = 2$	$\frac{4}{2} = 2$	$\frac{6}{3} = 2$	$\frac{8}{4} = 2$	$\frac{10}{5} = 2$	$\frac{12}{6} = 2$
$\frac{3}{1} = 3$	$\frac{6}{2} = 3$	$\frac{9}{3} = 3$	$\frac{12}{4} = 3$	$\frac{15}{5} = 3$	$\frac{18}{6} = 3$
$\frac{4}{1} = 4$	$\frac{8}{2} = 4$	$\frac{12}{3} = 4$	$\frac{16}{4} = 4$	$\frac{20}{5} = 4$	$\frac{24}{6} = 4$
$\frac{5}{1} = 5$	$\frac{10}{2} = 5$	$\frac{15}{3} = 5$	$\frac{20}{4} = 5$	$\frac{25}{5} = 5$	$\frac{30}{6} = 5$
$\frac{6}{1} = 6$	$\frac{12}{2} = 6$	$\frac{18}{3} = 6$	$\frac{24}{4} = 6$	$\frac{30}{5} = 6$	$\frac{36}{6} = 6$
$\frac{7}{1} = 7$	$\frac{14}{2} = 7$	$\frac{21}{3} = 7$	$\frac{28}{4} = 7$	$\frac{35}{5} = 7$	$\frac{42}{6} = 7$
$\frac{8}{1} = 8$	$\frac{16}{2} = 8$	$\frac{24}{3} = 8$	$\frac{32}{4} = 8$	$\frac{40}{5} = 8$	$\frac{48}{6} = 8$
$\frac{9}{1} = 9$	$\frac{18}{2} = 9$	$\frac{27}{3} = 9$	$\frac{36}{4} = 9$	$\frac{45}{5} = 9$	$\frac{54}{6} = 9$
$\frac{7}{7} = 1$	$\frac{8}{8} = 1$	$\frac{9}{9} = 1$	$\frac{10}{10} = 1$	$\frac{11}{11} = 1$	$\frac{12}{12} = 1$
$\frac{14}{7} = 2$	$\frac{16}{8} = 2$	$\frac{18}{9} = 2$	$\frac{20}{10} = 2$	$\frac{22}{11} = 2$	$\frac{24}{12} = 2$
$\frac{21}{7} = 3$	$\frac{24}{8} = 3$	$\frac{27}{9} = 3$	$\frac{30}{10} = 3$	$\frac{33}{11} = 3$	$\frac{36}{12} = 3$
$\frac{28}{7} = 4$	$\frac{32}{8} = 4$	$\frac{36}{9} = 4$	$\frac{40}{10} = 4$	$\frac{44}{11} = 4$	$\frac{48}{12} = 4$
$\frac{35}{7} = 5$	$\frac{40}{8} = 5$	$\frac{45}{9} = 5$	$\frac{50}{10} = 5$	$\frac{55}{11} = 5$	$\frac{60}{12} = 5$
$\frac{42}{7} = 6$	$\frac{48}{8} = 6$	$\frac{54}{9} = 6$	$\frac{60}{10} = 6$	$\frac{66}{11} = 6$	$\frac{72}{12} = 6$
$\frac{49}{7} = 7$	$\frac{56}{8} = 7$	$\frac{63}{9} = 7$	$\frac{70}{10} = 7$	$\frac{77}{11} = 7$	$\frac{84}{12} = 7$
$\frac{56}{7} = 8$	$\frac{64}{8} = 8$	$\frac{72}{9} = 8$	$\frac{80}{10} = 8$	$\frac{88}{11} = 8$	$\frac{96}{12} = 8$
$\frac{63}{7} = 9$	$\frac{72}{8} = 9$	$\frac{81}{9} = 9$	$\frac{90}{10} = 9$	$\frac{99}{11} = 9$	$\frac{108}{12} = 9$

72. Second sign of Division, what is it? Third mode of indicating Division, what is it? Fourth method, what? **73.** How are the dividend and divisor written in the second Division Table?

Ex. 31. How many are $\frac{24}{6}$, or 24 $\div$ 6? Ans. 4.

32. How many are $\frac{35}{5}$, or 35 $\div$ 5? $\frac{32}{8}$, or 32 $\div$ 8?

33. How many are $\frac{18}{2}$, or 18 $\div$ 2? $\frac{28}{7}$, or 28 $\div$ 7?

34. How many are $\frac{42}{6}$, or 42 $\div$ 6? $\frac{49}{7}$, or 49 $\div$ 7?

35. How many are $\frac{63}{9}$, or 63 $\div$ 9? $\frac{72}{8}$, or 72 $\div$ 8?

WRITTEN EXERCISES.

74. To perform Short Division.

Ex. 1. If 7 days make a week, how many weeks are there in 364 days?

OPERATION.

Divisor, 7) 3 6 4 Dividend.

Quotient, 5 2

Having written the divisor and dividend as in the margin, we first say, 7 in 36, 5 times and 1 remainder; set the quotient, 5, under the 6 of the dividend, and then *imagine* the remainder, 1, placed *before* the 4, and say, 7 in 14, 2 times; set the 2 under the 4, and thus we find the quotient to be, 52.

75. This process, called *Short Division*, usually employed when the divisor is small, may be performed by the following

RULE. *Having set the divisor at the left of the dividend with a line between them, divide the fewest figures in the left of the dividend that will contain the divisor, and set the quotient under the right hand figure taken in the dividend; if anything remains, prefix it* MENTALLY *to the next figure in the dividend, and divide the number thus formed as before, and so proceed till all the figures of the dividend have been employed.*

---

75. When is Short Division usually employed? Rule? How are divisor and dividend written? Which figures of the dividend are used first? How many? Quotient, where set? Remainder, to what is it prefixed? How? What is done with the number so formed? How far is the process carried?

	Ex. 2.	3.	4.
Divisor,	6) $351.54 Dividend.	5) 875	4) 568
Quotient,	$58.59		142

	5.	6.	7.	8.	9.
Divide	7248.	8127.	6345.	3528.	2576.
By	8.	3.	5.	9.	7.

76. When there is *no remainder*, as in the first nine examples, the division is *complete.* The dividend is then said to be divisible by the divisor, and the divisor is called an *exact divisor.* When there is *a remainder*, as in Ex. 10, the division is *incomplete*, and the dividend is said to be *indivisible* by the divisor.

10. Divide 325 by 7. Ans. $46\frac{3}{7}$.

OPERATION.

Divisor, 7) 3 2 5 Dividend.

Quotient, 4 6 .. 3 Remainder.

NOTE 1. The remainder is often written over the divisor in the fractional form (Art. 73), and the fraction is annexed to the quotient; thus, the answer in Ex. 10 is written $46\frac{3}{7}$ which, expressed in words, is forty-six and *three-sevenths.* Other fractions are read in a similar manner; thus, $\frac{1}{2}$ means *one-half;* $\frac{1}{3}$ *one-third;* $\frac{2}{3}$ *two-thirds;* $\frac{5}{9}$ *five-ninths;* etc.

NOTE 2. The remainder, when written in a fractional form as a part of the quotient, *becomes* like the quotient.

11. Divide 6276 by 5. Ans. 1255, and 1 remainder.
12. Divide 8765 by 5.
13. Divide 4823 by 8.
14. Divide 6358 by 6.
15. Divide 7296 by 2.
16. Divide 2594 by 3.
17. Divide 7828 by 4.
18. Divide 7358 by 7.
19. Divide 8454 by 9.
20. Divide $684 by 4.
21. Divide $6.84 by 4.
22. Divide $985 by 5.
23. Divide $9.85 by 5.

---

76. When is the division complete? When is one number *divisible* by another? What is an exact divisor? When is one number *indivisible* by another? How is the remainder often written? The fraction where placed?

24. How many pounds of sugar, at 9 cents per pound, can be bought for $35.64? Ans. 396.

25. How many barrels of flour, at $ 8 a barrel, can be bought for $ 5368?

26. If 6 shillings make a dollar, how many dollars are there in 7416 shillings? Ans. 1236.

27. If 4 weeks make a month, how many months are there in 3716 weeks?

28. How many oranges, at 6 cents apiece, can you buy for $35.64? Ans. 594.

29. If 7328 marbles are divided equally among 8 boys, how many marbles will each boy receive? Ans. 916.

30. If an estate, worth $16,492 dollars, is divided equally among 7 children, how many dollars does each child receive?

31. Divide two thousand one hundred and forty-two by six.

32. A mile is equal to 5280 feet; how many steps of 3 feet each will a man take in walking a mile?

77. To perform Long Division:

33. Divide 4654 by 13. Ans. 358.

OPERATION.

```
13)4654(358
   39
   ---
    75
    65
    ---
    104
    104
    ---
      0
```

Having set the divisor and dividend as in Short Division, draw a curve at the right of the dividend, and then say, 13 in 46, 3 times, and set the 3 at the right of the dividend. Then multiply the divisor by the quotient, 3, and set the product, 39, under the 46 of the dividend, and subtract the 39 from the 46. To the remainder, 7, annex 5, the next figure of the dividend, so forming a new partial dividend, 75, and then say, 13 in 75, 5 times, and set the 5 as the next figure of the quotient. Multiply the divisor by

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77. Explain Ex. 33. Of what *order* is any quotient figure? Illustrate.

this new quotient-figure, and subtract the product from the partial dividend. Proceed in this manner until the whole dividend has been divided; the entire quotient is 358.

Every quotient-figure is of the same order as the right-hand figure of the dividend used in obtaining that quotient-figure; thus in Ex. 33, the 46 of the dividend is *hundreds*, and the 3 of the quotient is *also hundreds;* the 75 is *tens* and the 5 of quotient is *also tens;* etc.

78. This process, called *Long Division*, usually employed when the divisor is large, may be performed by the following

RULE 1. *Write the divisor and dividend as in Short Division.*

2. *Divide the smallest number of figures in the left of the dividend that will contain the divisor, and set the result as the first figure of the quotient at the right of the dividend.*

3. *Multiply the divisor by the quotient-figure, and set the product under that part of the dividend taken.*

4. *Subtract the product from the figures over it, and to the remainder annex the next figure of the dividend for a new partial dividend.*

5. *Divide, and proceed as before, until the whole dividend has been divided.*

NOTE 1. It will be seen that the process of dividing consists of four distinct steps, viz.: first, to seek a quotient figure; second, multiply; third, subtract; and, fourth, form a new partial dividend by annexing the next figure of the dividend to the remainder.

NOTE 2. If any partial dividend will not contain the divisor, 0 must be placed in the quotient, and another figure brought down and annexed to the dividend.

---

78. When is Long Division employed? Give the rule for Long Division. How many steps in dividing? What are they? Repeat Note 2.

Note 3. If the product of the divisor multiplied by the quotient figure is greater than the partial dividend, the quotient figure is too large, and must be diminished.

Note 4. If the remainder equals or exceeds the divisor, the quotient is too small, and must be increased.

79. In the same manner solve the following examples, dividing each upper number by the one under it in each example; also, in the same manner, as suggested by the *signs.*

34. { Divide 18564 ÷ 156. Ans. 119.
  By 1092 ÷ 12.
35. { Divide 24453 ÷ 143. Ans. 171.
  By 1287 ÷ 11.
36. { Divide 20995 ÷ 221. Ans. 95.
  By 1105 ÷ 13.
37. { Divide 143405 ÷ 989. Ans. 145.
  By 4945 ÷ 23.

80. *Division is the reverse of multiplication.* In multiplication, the two factors are given, and the product is required; in division the product and one factor are given, and the other factor is required. The dividend is the product, and the divisor and quotient are the factors; thus,

IN MULTIPLICATION.		IN DIVISION.		
Factors,	Product.	Dividend,	Divisor,	Quotient.
5 × 4 =	20	20 ÷	5 =	4
		Or, 20 ÷	4 =	5

Hence the following

Proof. *Multiply the divisor by the quotient, and to the product add the remainder; the* SUM *should be the dividend.*

---

78. Repeat Note 3. Note 4. **80.** What is said of Division and Multiplication? In Multiplication what is given? What required? In Division what is given? Required? How is Division proved?

38. Divide 2537 by 53.

OPERATION.

53)2537(47
212
417
371
Remainder, 46

PROOF.

53 Divisor.
47 Quotient.
371
212
46 Remainder.
2537 Dividend.

39.

43)87349(2031
86
134
129
59
43
Remainder, 16

40.

47)943454(20073
94
345
329
164
141
Remainder, 23

	Quotients.	Rem.
41. Divide 6384 by 79.	80,	64.
42. Divide 7639 by 83.	92,	3.
43. Divide 18805 by 37.		
44. Divide 116092 by 29.	4003,	5.
45. Divide 47065 by 231.		
46. Divide 29768 by 123.	242,	2.
47. Divide 17693 by 149.		
48. Divide 98074 by 483.	203,	25.
49. Divide 69847 by 348.		

50. A farm containing 327 acres, was bought for $ 37605; what was the price per acre? Ans. $ 115.

51. Divide six thousand eight hundred and forty-four acres of land into twenty-nine equal parts. Ans. 236 acres.

52. A drover paid $2331 for 37 oxen; what was the average price per ox? Ans. $63.

53. The product of two numbers is 35068765, and one of the numbers is 8765; what is the other number? Ans. 4001.

54. In how many days will a steamboat sail 11352 miles, if she sails 264 miles per day?

55. If a railroad 359 miles long cost $3545484, what was the average cost per mile? Ans. $9876.

CONTRACTIONS.

81. To divide by a composite number.

56. Divide $1855 equally among 35 men.

OPERATION.

35 = 7 × 5.
1st Factor, 7) $1 8 5 5 Dividend.
2d Factor, 5) $2 6 5 1st Quotient.
$5 3 True Quotient.

The 35 men may be separated into 7 groups of 5 men each. Then dividing by 7 gives $265 for each group, and dividing the $265 by 5 gives $53 for each man.

NOTE. When a composite number is made up of different sets of factors, as in Ex. 57, it is immaterial which set is taken. It is also immaterial in what order the factors are taken.

---

81. Rule for dividing by a composite number? Is it material which factor of the divisor is used first, or which set of factors is employed?

57. Divide 10656 by 288.

288 = 4 × 6 × 12 = 6 × 6 × 8 = 8 × 3 × 12, etc.

FIRST OPERATION.		SECOND OPERATION.
4) 1 0 6 5 6		6) 1 0 6 5 6
6) 2 6 6 4		6) 1 7 7 6
1 2) 4 4 4		8) 2 9 6
3 7 Ans.	=	3 7 Ans.

From these examples we have the following

Rule. *Divide the dividend by one factor of the divisor, and the quotient so obtained by another factor, and so on till all the factors of the set have been used. The last quotient will be the true quotient.*

58.	Divide 20088 by 24; = 6 × 4.	Ans. 837.
59.	Divide 8445 by 15.	
60.	Divide 23296 by 32.	Ans. 728.
61.	Divide 26568 by 72.	
62.	Divide 22720 by 64.	Ans. 355.
63.	Divide 33696 by 144; = 12 × 12.	

82. In dividing by the factors of the divisor, there may be a *remainder*, after either or each of the divisions.

Should the learner find a difficulty in determining the *true remainder*, he has but to remember that it is *always* of the same kind as the dividend. (Art. 69, Note).

64. Divide 86 by 21; = 7 × 3.

OPERATION.

7) 8 6

3) 1 2 . . 2 Rem.

Quotient, 4

In this example, as 86 is the true dividend, 2 is the true remainder.

---

82. Rule for finding the true remainder when the factors of the divisor are used separately? The reason?

65. Divide 92 by 28; = 4 × 7.

OPERATION.
4) 9 2
——
7) 2 3
——
Quotient, 3 .. 2 Rem.

In this example, as 23 is only one-fourth of the true dividend, so the remainder, 2, is only one fourth of the true remainder; therefore the true remainder is 2 × 4 = 8.

66. Divide 527 by 42; = 6 × 7.

OPERATION.
6) 5 2 7
——
7) 8 7 .. 5 Rem.
——
Quotient, 1 2 .. 3 Rem.

From the previous explanations we see that 5 our first remainder here is one part of the true remainder, and that 3, the second remainder, multiplied by 6, the first divisor, is the other part; that is, 5 + 3 × 6 = 23; is the true remainder. Similar reasoning applies when there are more than two divisors. Hence,

To obtain the true remainder when division is performed by using the factors of the divisor:

Rule. *Multiply each remainder, except that left by the first division, by the continued product of the divisors preceding that which gave the remainders severally, and the sum of the products, together with the remainder left by the first division, will be the true remainder.*

Note. When there are but two divisors and two remainders, it only requires the addition of the *first remainder* to the *product* of the *first divisor and second remainder.*

67. Divide 1834 by 35; = 5 × 7. Ans. Quo. 52, Rem. 14.

68. Divide 15008 by 315; = 5 × 7 × 9.
Ans. Quo. 47, Rem. 203.

69. Divide 7704 by 105; = 3 × 5 × 7.

70. Divide 45621 by 405; = 5 × 9 × 9.
Ans. Quo. 112, Rem. 261.

71. Divide 55242 by 25.

83. To divide by 10, 100, 1000, etc.

Rule. *Cut off by a point, as many figures from the right hand of the dividend as there are ciphers in the divisor. The figures at the left of the point are the quotient, and those at the right are the remainder.*

72. Divide 756 by 10. Ans. 75.6, = 75 Quo. and 6 Rem.

Note. The reason of the rule is obvious. By taking away the right-hand figure, each of the other figures is brought one place nearer to units, and its value is only one-tenth as great as before (Art. 18), and therefore the whole is divided by 10. For like reasons, cutting off *two* figures divides by 100; cutting off *three* figures divides by 1000, etc.

73. Divide 4867 by 100. Ans. Quo. 48, Rem. 67.
74. Divide 37692 by 1000.
75. Divide 5367842 by 1000000.
76. Divide 20687432004 by 1000000000.

84. To divide by 20, 50, 700, or any like number:

Rule. *Cut off as many figures from the right of the dividend as there are ciphers at the right of the significant figures of the divisor, and then divide the remaining figures of the dividend by the significant figures of the divisor.*

Note 1. This is on the principle of dividing by the factors of the divisor; therefore the true remainder will be found by the rule in Art. 82.

77. Divide 74689 by 8000. Ans. 9 Quo. and 2689 Rem.

OPERATION.

8) 74.689

Quotient, 9 . . . 2 Rem.

We divide by 1000 by cutting off 689, which gives 74 for a quotient and 689 for a remainder; then divide 74 by 8, and obtain the quotient, 9, and remainder, 2. This remainder, 2, is 2000, which, increased by 689, gives 2689 for the true remainder (Art. 82).

---

83. Rule for dividing by 10? By 100? Reason of rule? **84.** Rule for dividing by 20? By 500? Reason? How is the true remainder found?

NOTE 2. It will be observed that the true remainder, in all examples like the 77th, is obtained by *annexing* the 1st to the 2d remainder.

78. Divide 3764 by 200. Ans. Quo., 18, Rem., 164.
79. Divide 4547 by 400.
80. Divide 3876423 by 7000. Ans. Quo. 553, Rem. 5423.
81. Divide 7943862 by 210000.

GENERAL PRINCIPLES OF DIVISION.

85. The value of a quotient depends upon the *relative* values of the divisor and dividend, and not upon their *absolute* values. This will be seen by the following propositions.

(1st). *If the divisor remains unaltered, multiplying the dividend by any number is, in effect, multiplying the quotient by the same number; thus,*

$$\begin{array}{rcr} 15 & \div\, 3 = & 5 \\ 4 & & 4 \\ \hline 60 & \div\, 3 = & 20 \end{array}$$

that is, multiplying the dividend by 4 multiplies the quotient by 4.

(2d). *Dividing the dividend by any number is dividing the quotient by the same number; thus,*

$$\begin{array}{rl} & 24 \div 2 = 12 \\ 3) & 24 \\ & \overline{8} \div 2 = 4 = 12 \div 3; \end{array}$$

that is, dividing the dividend by 3 divides the quotient by 3.

---

85. Does the size of the quotient depend upon the absolute size of the divisor and dividend? Upon what does it depend? What is the first proposition? Second? Third? Fourth?

(3d). *Multiplying the divisor by any number divides the quotient by the same number; thus,*

$$\begin{array}{rcl} 30 \div & 2 & = 15 \\ & 3 & \\ \cline{2-2} 30 \div & 6 & = 5 = 15 \div 3; \end{array}$$

that is, multiplying the divisor by 3 divides the quotient by 3.

(4th). *Dividing the divisor by any number multiplies the quotient by the same number; thus,*

$$\begin{array}{rcl} 40 \div & 10 & = 4 \\ & 5)\,10 & \\ \cline{2-2} 40 \div & 2 & = 20 = 4 \times 5; \end{array}$$

that is, dividing the divisor by 5 multiplies the quotient by 5.

(5th). It follows, from (1st) and (2d), *that the greater the dividend the greater is the quotient; and the less the dividend the less the quotient.*

(6th). Also, from (3d) and (4th), *that the greater the divisor, the less is the quotient; and the less the divisor the greater the quotient.*

86. From the illustrations in Art. 85 we see that any change in the *dividend* causes a SIMILAR change in the quotient, and that any change in the *divisor* causes an OPPOSITE change in the quotient. Hence,

(1st). *Multiplying both dividend and divisor by the same number does not affect the quotient; thus,*

$$\begin{array}{ccccl} 12 & \div & 3 & = & \\ 2 & & 2 & & \\ \cline{1-1}\cline{3-3} 24 & \div & 6 & = & 4, \text{ Quotient unchanged.} \end{array}$$

---

85. What follows from (1st) and (2d)? What follows from (3d), (4th)? From (5th), (6th)? 86. Any change in the dividend, how affects the quotient? Any change in the divisor, how? First inference? Second? Third? Illustrate.

(2d). *Dividing both dividend and divisor by the same number does not affect the quotient;* thus,

$$\begin{array}{rcl} 20 & \div & 10 = 2 \\ 5)\underline{20} & & 5)\underline{10} \\ 4 & \div & 2 = 2, \text{ Quotient unchanged.} \end{array}$$

(3d). It follows, from (1st) and (2d), *that the operations of multiplying and dividing by the same number cancel, that is destroy, each other;* for example,

If a number be multiplied by any number, and the product be divided by the multiplier, the quotient will be the multiplicand; thus,

$8 \times 7 = 56$ and $56 \div 7 = 8$, the multiplicand.

Also, if a number be divided by any number, and the quotient be multiplied by the divisor, the product will be the dividend; thus,

$15 \div 3 = 5$, and $5 \times 3 = 15$, the dividend.

87. These general principles may be more briefly stated as follows:

(1st). *Multiplying the dividend multiplies the quotient; and dividing the dividend divides the quotient* (Art. 85, 1*st* and 2*nd*).

(2d). *Multiplying the divisor divides the quotient; and dividing the divisor multiplies the quotient* (Art. 85, 3*d* and 4*th*).

(3d). *Multiplying both dividend and divisor by the same number; or dividing both by the same number, does not affect the quotient* (Art. 86, 1*st* and 2*d*).

---

87. A more brief statement of these principles: First? Second? Third?

CANCELLATION.

88. How many are 6 times 7 divided by 6?

OPERATION.

$$\frac{\cancel{6}\times 7}{\cancel{6}} = 7,\ \text{Ans.}$$

Indicating the multiplication and division (Art. 73), we may *cancel* or *strike out* the equal factors, 6 and 6, from the divisor and dividend, and so shorten the process without changing the result.

How many are 7 times 12 divided by 6?

OPERATION.

$$\frac{7\times 12}{6}\ \text{or,}\ \frac{7\times 2\times \cancel{6}}{\cancel{6}} = 14\ \text{Ans.}$$

Separating the 12 into the two factors, 2 and 6, cancel the 6 from divisor and dividend, and there is left 7 times 2, equal to 14, for the quotient.

Or,

$$\frac{7\times \overset{2}{\cancel{12}}}{\cancel{6}} = 14,\ \text{Ans.}$$

This process is, in effect, the same as the other. Instead of resolving 12 into the factors, 2 and 6, we merely divide 12 by 6, setting the quotient, 2, over the 12; then, cancelling the 6 and 12, the result is 7 times 2, equal to 14, as before.

How many are 8 times 15 divided by 12?

OPERATION.

$$\frac{\overset{2}{\cancel{8}}\times \overset{5}{\cancel{15}}}{\underset{\cancel{3}}{\cancel{12}}} = 10,\ \text{Ans.}$$

First, reject or cancel the factor 4 from both 8 and 12, giving 2, which is placed over 8 and 3, placed under 12; then cancel 3 in 3, and in 15, giving 5 to place over 15, and we have 2 times 5, equal to 10, for the result.

These examples are solved, in part, by cancelling. Hence,

89. Cancellation is a process for shortening the solution

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89. What is Cancellation? On what principle does it depend?

of an example, by rejecting, or *cancelling* the same factors from both divisor and dividend.

It depends on the principle (Art. 86, *2d*), that *dividing both dividend and divisor by the same number does not affect the quotient.*

Ex. 1. Divide 8 × 3 × 10 × 63 by 4 × 5 × 7.

OPERATION.

$$\frac{\overset{2}{\cancel{8}} \times 3 \times \overset{2}{\cancel{10}} \times \overset{9}{\cancel{63}}}{\cancel{4} \times \cancel{5} \times \cancel{7}} = 108, \text{ Ans.}$$

We cancel 4 in 8, giving 2; 5 in 10 giving 2, and 7 in 63, giving 9. Then, 2 × 3 × 2 × 9 = 108, the Ans.

2. Divide 6 × 21 × 15 × 11 by 18 × 7 × 5. Ans. 33.

3. Divide 9 × 14 × 26 × 8 by 3 × 21 × 13 × 4.

4. How many cords of wood, at $6 a cord, will pay for 5 tons of hay, at $12 a ton?

5. How many tons of hay, at $15 a ton, will pay for 4 acres of land, at $ 45 an acre?

6. How many pounds of butter, at 33 cents a pound, must be given for 3 boxes of raisins, each containing 22 pounds, at 15 cents a pound? Ans. 30.

7. How many pieces of cloth, containing 32 yards each, at $ 3 per yard, will pay for 48 barrels of flour, at $12 per barrel? Ans. 6.

8. How much sugar, at 15 cents a pound, will be required to pay for 3 boxes of lemons, containing 305 lemons each, at 4 cents apiece: Ans. 244.

9. How many boxes of tea, each containing 45 pounds, at 66 cents a pound, must be given for 15 boxes of sugar, each containing 220 pounds, at 18 cents a pound?

10. How many bags of corn, each containing 2 bushels, at 96 cents a bushel, will pay for 128 bags of oats, each containing 3 bushels, at 47 cents a bushel? Ans. 94.

REVIEW AND TEST QUESTIONS.

90. Let the pupil answer the following questions, illustrating them with his own original examples:

1. How will you find the sum of several given numbers?

2. How will you find the difference between two given numbers?

3. How will you find the subtrahend when the minuend and remainder are given?

4. How will you find the minuend when the subtrahend and remainder are given?

5. How will you find the remainder, when the minuend and subtrahend are given?

6. When the sum of several numbers and all of the numbers except one are given, how do you find that one?

7. When the multiplicand and multiplier are given, how can you find the product?

8. When the multiplicand and product are given, how can you find the multiplier?

9. When the multiplier and product are given, how can you find the multiplicand?

10. How do you find the quotient, when the dividend and divisor are given?

11. How do you find the divisor, when the dividend and quotient are given?

12. How do you find the dividend, when the divisor and quotient are given?

13. How do you find the dividend, when the quotient, divisor, and remainder are given?

14. How do you find the divisor, when the dividend, quotient and remainder are given?

EXAMPLES IN THE FOREGOING PRINCIPLES.

1. A boy sold a sled for $ 2.00 and in payment received 50 cents in money, 5 pineapples at 20 cents each, and the remainder in oranges at 5 cents each; how many oranges did he receive? Ans. 10.

2. If 2 barrels of flour will pay for 5 yards of broadcloth, how many barrels of flour will pay for 3 times 5 yards of broadcloth?

3. How many barrels of apples, at $ 3 a barrel, must be given for 6 yards of flannel, at $ 2 a yard?

4. A speculator bought 80 acres of land at $ 75 per acre, and sold the whole for $ 6720; how much did he gain by the transactions? How much per acre? First Ans. $ 720.

5. Bought 160 acres of land for $ 4000, and sold it at $ 20 per acre; did I gain or lose? How much? How much per acre?

6. If 2 men build 16 rods of wall in 4 days, in how many days will 5 men build 50 rods? Ans. 5.

7. How many miles per hour must a steamboat sail to cross the Atlantic, 2880 miles, in 10 days of 24 hours each? Ans. 12.

8. The product of two factors is 595; one of the factors is 17; what is the other?

9. The product of three factors is 9177; two of the factors are 21 and 19; what is the other? Ans. 23.

10. The divisor is 18, and the quotient 13; what is the dividend?

11. The divisor is 23, the quotient 37, and the remainder 19; what is the dividend? Ans. 870.

12. The first of three numbers is 8, the second is 4 times the first, and the third is 3 times the sum of the other two; what is the difference between the first and third? Ans. 112.

13. In how many days of 24 hours each, will a ship cross the Atlantic, 2880 miles, if she sails 12 miles per hour?

14. If I receive $ 80 and spend $ 55 per month, in how many years of 12 months each shall I save $ 1800?

15. Bought 87 yards of cloth, at $ 4 per yard, and paid $ 200 in money and the rest in wheat at $ 2 per bushel; how many bushels of wheat did it take? Ans. 74.

16. The subtrahend is 3762, and the remainder is 2657; what is the minuend?

17. The minuend is 8063, and the remainder is 5604; what is the subtrahend? Ans. 2459.

18. The greater of two numbers is 8327, and the difference is 5364; what is the less number? Ans. 2963.

19. The sum of two numbers is 5836, and the less number is 2467; what is the difference between the two numbers?

20. A man having engaged to work 12 months for $ 432, left his employer at the end of 9 months; at the rate agreed upon, what should he receive? Ans. $ 324.

21 A merchant received $ 376 on Monday, $ 567 on Tuesday, $ 487 on Wednesday, $ 684 on Thursday, $ 293 on Friday, and $ 857 on Saturday; what were his average receipts per day?

22. If 732 is multiplied by 27 and the product divided by 36, what is the quotient? Ans. 549.

23. Bought 175 barrels of flour for $ 1750, and sold 86 barrels of it at $ 12 a barrel, and the remainder at $ 9 a barrel; did I gain or lose? How much?

24. How many are $376 + 874 + 563 - 937$?

25. How many are $384 + 562 + 1728 \div 191$?

DENOMINATE NUMBERS AND REDUCTION.

91. All numbers are either *concrete* or *abstract.*

A CONCRETE NUMBER is one that is applied to a particular object; as 6 books, 4 men, 7 days, 3 rods. A concrete number is often called a *Denominate Number,* because it *denominates or names* the thing to which it is applied.

AN ABSTRACT NUMBER is one that is *not* applied to any particular object; as 6, 9, 23.

92. All numbers are either *simple* or *compound.*

A SIMPLE NUMBER consists of but *one kind,* and may be *abstract* or *concrete;* as 2, $4, 10 miles, 3 pounds.

A COMPOUND NUMBER consists of *two or more denominations,* and is *necessarily concrete;* as 4 days and 7 hours; 3 pecks, 5 quarts, and 1 pint; 8 rods, 4 yards, 2 feet, and 10 inches.

NOTE 1. The several parts of a compound number, though of *different denominations,* are yet of the *same general nature;* thus, 2 weeks, 3 days, and 6 hours are SIMILAR *quantities, and constitute a compound number;* but 2 weeks, 3 miles, and 6 quarts are UNLIKE IN THEIR NATURE, *and do* NOT *constitute a compound number.*

NOTE 2. The first division of each of the following tables should be *thoroughly committed to memory.* The second division is designed for *reference.*

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91. What is a Concrete Number? What is it often called? Why? What is an Abstract Number?

92. What is a Simple Number? May it be abstract? Concrete? Illustrate. What is a Compound Number? May it be abstract? Illustrate. What is said of the different denominations of a compound number? Is *this* a compound number: 3 rods, 2 pecks, and 5 pounds? Why? What is said of the first division of the following tables? What of the second?

ENGLISH MONEY.

93. **English Money** is the Currency of Great Britain.

TABLE.

4 Farthings (far. or qr.) make	1 Penny,	marked	d.
12 Pence "	1 Shilling,	"	s.
20 Shillings "	1 Pound,	"	£

£		s.		d.		qr.
				1	=	4
		1	=	12	=	48
1	=	20	=	240	=	960

Note. The numbers employed as multipliers and divisors in reducing a Compound Number are called a *Scale;* thus, in reducing English Money, the *Descending* Scale is 20, 12, and 4; and the *Ascending* Scale is 4, 12, and 20.

Ex. 1. In 7£ 1s. 6d. 3qr. how many farthings?

OPERATION.

£	s.	d.	qr.
7	1	6	3

2 0
——
1 4 1 s.
1 2
——
1 6 9 8 d.
4
——
6 7 9 5 qr., Ans.

Multiply the 7 by 20 to change the pounds to shillings; to the product, 140, add the 1s. given in the example, and the result is 141s.; then multiply the 141 by 12 to change the shillings to pence; to the product, 1692, add the 6d. in the example, and the result is 1698d.; so proceed till the example is solved.

---

93. What is English Money? Repeat the Table. What are the multipliers and divisors used in reducing a compound number called? What is the descending scale in English Money? What the ascending scale? Explain Example 1. Explain Ex. 2.

Ex. 2. Change 6795 qr. to pence, shillings, and pounds.

OPERATION.

4) 6 7 9 5 qr.

12) 1 6 9 8 d. + 3qr.

20) 1 4 1 s. + 6d.

7 £ + 1s.

First divide by 4 to change the farthings to pence, giving 1698d. and 3qrs.; then divide the 1698 by 12 to change the pence to shillings, giving 141s. and 6d.; then divide the 141 by 20 to change the shillings to pounds, giving 7£ 1s., and thus obtain 7£ 1s. 6d. 3qr., Ans.

These examples are questions in *Reduction*. Hence,

94. REDUCTION consists in changing a number of one denomination to a number of another denomination, *without changing its value.*

The process in Ex. 1 is called *Reduction Descending*, because *higher denominations are changed to lower.* Hence,

95. REDUCTION DESCENDING consists in changing a number from a *higher* to a *lower* denomination, and may be performed by the following

RULE. *Multiply the highest denomination given, by the number it takes of the next lower denomination to make one of this higher, and to the product add the number of the lower denomination; multiply this sum by the number it takes of the* NEXT *lower denomination to make one of* THIS; *add as before, and so proceed till the number is brought to the denomination required.*

96. The process in Ex. 2 is called *Reduction Ascending*, because *lower denominations are changed to higher.* Hence,

---

94. What is Reduction? What is the process in Ex. 1 called? Why? **95.** What is Reduction Descending? Rule for performing it? **96.** What is the process in Ex. 2 called? Why?

REDUCTION ASCENDING consists in changing a number from a *lower* to a *higher* denomination, and may be performed by the following

RULE. *Divide the given number by the number it takes of that denomination to make one of the next higher; divide the quotient by the number it takes of* THAT *denomination to make one of the* NEXT *higher, and so proceed till the number is brought to the denomination required. The last quotient, together with the several remainders* (Art. 69, Note), *will be the answer.*

97. The processes in Reduction Ascending and Reduction Descending *prove* each other, as will be seen in Examples 3 and 4. In the same manner let the pupil prove all the examples in Reduction, and the answers, for this purpose, will be omitted in the book.

Ex. 3. How many farthings in 16£ 3s. 9d. 2qr.?

OPERATION.

16£ 3s. 9d. 2qr.
20
———
323 s.
12
———
3885 d.
4
———
15542 qr., Ans.

Ex. 4. Reduce 15542 qr. to pence, shillings, etc.

OPERATION.

4)15542 qr.
———
12)3885 d. + 2qr.
———
20)323 s. + 9d.
———
16 £ + 3s.

Ans. 16£ 3s. 9d. 2qr.

NOTE 1. In solving Ex. 3, the several numbers of the lower denominations are added *mentally*, and only the *results are written*; thus, 20 times 16 are 320, and the 3s. added give 323s. Then mul-

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96. What is Reduction Ascending? Rule for performing it? 97. How are processes in Reduction proved? In solving Ex. 3, how are the numbers of the lower denominations added?

tiplying the 323 by 12, and adding the 9d., we have 3885d. Finally, multiplying the 3885 by 4, and adding the 2qr., we have 15542qr. which is the Ans.

Note 2. In solving Ex. 4, and other examples in Reduction Ascending, if any divisor is so large that the work is not easily done by Short Division, the numbers may be taken upon the slate and the work done by Long Division.

5. Reduce 27£ 16s. 11d. 1qr. to farthings.
6. Reduce 17375 qrs. to pence, shillings, and pounds.
7. Reduce 54£ 9s. 3qr. to farthings.
8. Reduce 25£ 3d. to farthings.
9. Reduce 12497qr. to higher denominations.
10. Reduce 23445 qr. to higher denominations.
11. A bookseller received from London fifty Oxford Bibles. The lot cost him 6£ 5s., how much was that apiece?

TROY WEIGHT.

98. Troy Weight is used in weighing gold, silver, and precious stones.

TABLE.

24 Grains (gr.)	make	1 Pennyweight,	dwt.
20 Pennyweights	"	1 Ounce,	oz.
12 Ounces	"	1 Pound,	lb.

				dwt.		gr.
		oz.		1	=	24
lb.		1	=	20	=	480
1	=	12	=	240	=	5760

Scale.—Descending, 12, 20, 24; ascending, 24, 20, 12.

---

97. In Ex. 4, how is the work done? **98.** For what is Troy Weight used? Repeat the table. What is the descending scale? Ascending? Explain Ex. 1.

Ex. 1. How many grains in 7lb. 11oz. 14dwt. 18gr.?

OPERATION.

7 lb. 11oz. 14dwt. 18gr.
12
95 oz.
20
1914 dwt.
24
7674
3828
45954 gr., Ans.

Ex. 2. Reduce 45954 gr. to pounds, ounces, etc.

OPERATION.

24) 45954 gr.
20) 1914 dwt. + 18gr.
12) 95 oz. + 14dwt.
7 lb. + 11oz.

Ans. 7lb. 11oz. 14dwt. 18gr.

NOTE 1. In reducing the *pennyweights to grains* in Ex. 1, we first multiply the 1914 by 4 and add the 18gr., giving 7674; then multiply the 1914 by the 2 tens, giving 3828 tens; and finally add the results, giving 45954gr., Ans.

3. Reduce 6lb. 4oz. 15dwt. 23gr. to grains.
4. Reduce 18lb. 11oz. 6dwt. 19gr. to grains.
5. Reduce 53649gr. to pennyweights, ounces, etc.
6. Reduce 63594gr. to higher denominations.
7. Reduce 15lb. 6dwt. to grains.
8. How many spoons, each weighing 2oz. 3dwt. 18gr., can be made from 1lb. 1oz. 2dwt. 12gr. of silver? Ans. 6.
9. A jeweller made 6oz. 7dwt. 12gr. of gold into rings, which weighed 3dwt. 13gr. each; how many rings did he make?

NOTE 2. In performing Exs. 8 and 9, and similar examples, *both* of the given quantities must *first* be reduced to the *lowest denomination mentioned in either.*

APOTHECARIES' WEIGHT.

99. Apothecaries' Weight is used in *mixing* or *compounding* medicines; but medicines are *bought* and *sold* by *Avoirdupois Weight.*

TABLE.

20 Grains (gr.)	make	1 Scruple,	sc. or ℈
3 Scruples	"	1 Dram,	dr. or ʒ
8 Drams	"	1 Ounce,	oz. or ℥
12 Ounces	"	1 Pound,	lb. or ℔

						sc.		gr.
				dr.		1	=	20
		oz.		1	=	3	=	60
lb.		1	=	8	=	24	=	480
1	=	12	=	96	=	288	=	5760

Scale. Descending, 12, 8, 3, 20; Ascending, 20, 3, 8, 12.

Note. The pound, ounce, and grain, in Apothecaries' and Troy Weight are equal, but the ounce is differently subdivided.

1. Reduce 2℔ 3℥ 5ʒ 1℈ 18gr. to grains.
2. Reduce 13298gr. to scruples, drams, etc.
3. In 5lb. 6oz. 5dr. 2sc. 14gr. how many grains?
4. In 3℔ 5℥ 33℈ 24gr. how many grains?
5. In 25437gr. how many scruples, drams, etc.?
6. Reduce 3764gr. to higher denominations.
7. What quantity of medicine will an apothecary use in preparing 365 prescriptions of 12 grains each?

Ans. 9oz. 1dr.

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99. For what is Apothecaries' Weight used? Repeat the table. Descending scale? Ascending? What denominations of Apothecaries' Weight are like those of Troy Weight? What of the ounce?

AVOIRDUPOIS WEIGHT.

100. Avoirdupois Weight is used in weighing the coarser articles of merchandise, such as hay, cotton, tea, sugar, copper, iron, etc.

TABLE.

16	Drams (dr.)	make	1 Ounce,	oz.
16	Ounces	"	1 Pound,	lb.
25	Pounds	"	1 Quarter,	qr.
4	Quarters	"	1 Hundred Weight,	cwt.
20	Hundred Weight	"	1 Ton,	t.

t.		cwt.		qr.		lb.		oz.		dr.
								1	=	16
						1	=	16	=	256
				1	=	25	=	400	=	6400
		1	=	4	=	100	=	1600	=	25600
1	=	20	=	80	=	2000	=	32000	=	512000

Scale. Descending, 20, 4, 25, 16, 16; Ascending, 16, 16, 25, 4, 20.

Note 1. It was the custom formerly to consider 28lb. a quarter, 112lb. a hundred weight, and 2240lb. a ton; but now the *usual* practice is in accordance with the table.

These different tons are distinguished as the *long* or *gross* ton = 2240lb. and the *short* or *net* ton = 2000lb.

The *gross* ton is still used in the wholesale coal trade; also in estimating goods at the U. S. custom-houses, etc.

Note 2. A pound in Avoirdupois Weight is equal to 7000 grains in Apothecaries, and Troy Weight.

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100. For what is Avoirdupois Weight used? Table? Scale? How many pounds now make a ton? How many formerly? What are the different tons called? For what is the long ton now used? One pound Avoirdupois equals how many grains Troy?

Ex. 1. Reduce 2t. 6cwt. 1qr. 23lb. to pounds.

OPERATION.

2t. 6cwt. 1qr. 23lb.
20

46 cwt.
4

185 qr.
25

925
370
23 lb.

4648 lb., Ans.

Ex. 2. In 4648lb. how many tons, etc.?

OPERATION.

25)4648 lb.

4)185 qr. + 23lb.

20)46 cwt. + 1qr.

2 t. + 6cwt.

Ans. 2t. 6cwt. 1qr. 23lb.

Note 3. Instead of *mentally* adding the numbers of the lower denominations, as in preceding examples and as is done with the 6cwt. and 1qr. in Ex. 1, the pupil may, if he chooses, write the numbers under the partial products, and then add, as is done with the 23lb. in this Example.

3. Reduce 5t. 7cwt. 3qr. 21lb. 15oz. 7dr. to drams.
4. Reduce 4t. 2qr. 15oz. to drams.
5. Reduce 147683dr. to higher denominations.
6. Reduce 1860861dr. to ounces, pounds, quarters, etc.
7. If a cow eats 16lb. of hay in 1 day, how many tons will she eat in 365 days?
8. In 7t. 16cwt. 3qr. 5lb. *net weight*, how many *gross tons*?

CLOTH MEASURE.

101. Cloth Measure is used in measuring cloths, ribbons, braids, etc.

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101. Explain Ex. 1. Explain Ex. 2. **101.** For what is Cloth Measure used? Table? Scale?

TABLE.

2¼ Inches (in.)	make	1 Nail,	na.
4 Nails	"	1 Quarter,	qr.
4 Quarters	"	1 Yard,	yd.

				na.		in.
		qr.		1	=	2¼
yd.		1	=	4	=	9
1	=	4	=	16	=	36

Scale. Descending, 4, 4, 2¼; Ascending, 2¼, 4, 4.

1. In 6yd. 2qr. 3na. how many nails?
2. In 107 nails how many quarters, etc.?
3. Reduce 18yd. 1qr. 2na. to nails.
4. Reduce 47yd. 3qr. 1na. to nails.
5. Reduce 783 nails to quarters, etc.
6. Reduce 549 nails to higher denominations.
7. If 2yd. 1qr. of ribbon are used in trimming 1 bonnet, how many yards will be used in trimming 5 bonnets?
8. If 2yd. 1qr. 3na. of cloth are used in making 1 coat, how many yards will be used in making 16 coats? Ans. 19.
9. How many dresses can be made from 117yd. 2qr. of silk, if each dress requires 14yd. 2qr. 3na. Ans. 8.
10. What cost 18yd. 3qr. of velvet, at $2 per quarter?

LONG MEASURE.

102. Long Measure is used in measuring distances; as, for example, the length of a line, or the length, breadth, height, or depth of any object.

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102 For what is Long Measure used? Table? Scale?

TABLE.

12 Inches (in.)	make	1 Foot,	ft.
3 Feet	"	1 Yard,	yd.
5½ Yards or 16½ Feet	"	1 Rod,	rd.
40 Rods	"	1 Furlong,	fur.
8 Furlongs	"	1 Mile,	m.
69½ Statute miles, nearly	"	1 Degree on Circ. of the Earth,	1°
360 Degrees	"	1 Circumference,	circ.

m.	fur.	rd.	yd.	ft.	in.
				1	= 12
			1	= 3	= 36
		1	= 5½	= 16½	= 198
	1	= 40	= 220	= 660	= 7920
1	= 8	= 320	= 1760	= 5280	= 63360

Scale. Descending, 360, 69½, 8, 40, 5½, 3, 12; Ascending, 12, 3, 5½, 40, 8, 69½, 360.

Note 1. The earth not being an exact sphere, the distance round it in different directions is not *exactly* the same. By the most exact measurements made, a degree is a little *less* than 69½ miles.

Note 2. Besides the numbers given in the table, there are various other measures of length; as, 3 barleycorns make 1 inch, 4 inches 1 hand, 9 inches 1 span, 3 feet 1 space, 6 feet 1 fathom, 3 geographic miles 1 league, 60 geographic miles 1 degree, etc.

1. How many rods in 5m. 6fur. 37rd.?
2. Reduce 1877 rods to higher denominations.
3. Reduce 3659 rods to higher denominations.
4. In 301 furlongs how many miles?
5. In 5yd. 1ft. 9in. how many inches?
6. In 197 inches how many feet, etc.?
7. The distance through the earth is about 7912 miles; how ny rods is it?

8. The distance round the earth is about 8000000 rods; how many miles is it? Ans. 25000.

9. The distance from the earth to the moon is about 240000 miles; how many rods is it?

10. The distance from the earth to the sun is about 30400000000 rods; how many miles is it?

CHAIN MEASURE.

103. Chain Measure is used by engineers and surveyors in measuring roads, canals, boundaries of fields, etc.

TABLE.

$7\frac{92}{100}$	Inches (in)	make	1	Link,	li.
25	Links	"	1	Rod, Perch, or Pole.	rd.
4	Rods	"	1	Chain,	ch.
10	Chains	"	1	Furlong,	fur.
8	Furlongs	"	1	Mile,	m.

								li.		in.
						rd.		1	=	$7\frac{92}{100}$
				ch.		1	=	25	=	198
		fur.		1	=	4	=	100	=	792
m.		1	=	10	=	40	=	1000	=	7920
1	=	8	=	80	=	320	=	8000	=	63360

Scale. Descending, 8, 10, 4, 25, $7\frac{92}{100}$; Ascending $7\frac{92}{100}$, 25, 4, 10, 8.

Note. To measure roads, etc., engineers often use a chain 100 feet long.

1. Reduce 3m. 4fur. 8ch. 2rd. 20li. to links.
2. Reduce 28870 links to higher denominations.
3. Reduce 5m. 7fur. 3ch. to links.

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102 A degree upon the earth, how long? What other measures of length?
103 For what is Chain Measure used? Table? Scale? Note?

4. Reduce 4m. 3ch. 22li. to links.

5. Reduce 35647 links to higher denominations.

6. The distance from Boston to Andover is about 184000 links; how many miles is it?

7. The distance round a field is 5fur. 7ch. 3rd.; what will it cost to fence this field at $ 2 per rod?

SQUARE MEASURE.

104. A SURFACE is that which has length and breadth but no thickness.

105. A four-sided figure having all its *corners or angles*, *equal* to each other, as A B C D, Fig. 1, is called a *Rectangle*,

FIG. 1.

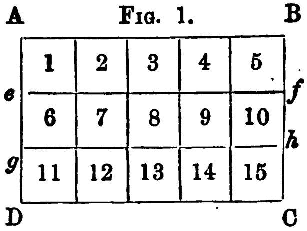
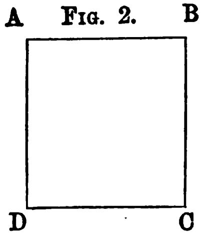


FIG. 2.



106. A *Rectangle* whose *sides* are all *equal* to each other, as A B C D, Fig. 2, is called a *Square*. The small checks in Fig. 1 are *squares*.

107. The manner of finding the *area* or *measure* of any rectangle, as, for example, Fig. 1, may be understood by the following explanation: — Let A B represent (on a reduced scale) a line five feet long; then, evidently, if we pass from A to *e*, a distance of 1 foot, and draw the line *e f* the figure A B *f e* will contain 5 square feet, that is 5 × 1 square feet. So, in

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104 What is a Surface? **105.** A Rectangle? **106.** A Square? **107.** How is the area of a Rectangle found?

like manner, A B *h g* will contain 10, or 5 × 2 square feet, and A B C D will contain 15, or 5 × 3 square feet. Hence we *multiply together the numbers representing the length and breadth of a rectangle to find its area.*

108. Reversing the process in Art. 107,

The area of a rectangle divided by its length will give its breadth, and the area divided by the breadth will give the length; thus, in Fig. 1, 15 ÷ 5 = 3, the *breadth,* and 15 ÷ 3 = 5, the *length.*

109. SQUARE MEASURE is used for measuring surfaces.

TABLE.

144	Square Inches (sq. in.)	make	1 Square Foot,	sq. ft.
9	Square Feet	"	1 Square Yard,	sq. yd.
30¼ 272¼	Square Yards or Square Feet	"	1 Square Rod,	sq. rd.
40	Square Rods	"	1 Rood,	r.
4	Roods	"	1 Acre,	a.
640	Acres	"	1 Square Mile,	sq. m.

(a) Also in CHAIN MEASURE,

10000 16	Square Links or Square Rods	make	1 Square Chain,	sq. ch.
10	Square Chains	"	1 Acre,	a.

sq. m.	a.	r.	sq. rd.	sq. yd.	sq. ft.	sq. in.
					1 =	144
				1 =	9 =	1296
			1 =	30¼ =	272¼ =	39204
		1 =	40 =	1210 =	10890 =	1568160
	1 =	4 =	160 =	4840 =	43560 =	6272640
1 =	640 =	2560 =	102400 =	3097600 =	27878400 =	4014489600

---

108. How is the breadth of a rectangle found when the area and length are known? How the length when the area and breadth are known?

109. For what is Square Measure used? Table? Scale? Table in Chain Measure? Note 1? Note 2?

SCALE. Descending, 640, 4, 40, 30¼, 9, 144; Ascending, 144, 9, 30¼, 40, 4,640.

NOTE 1. In measuring land, surveyors use a 4-rod chain composed of 100 links. Sometimes the half-chain of 50 links is used.

NOTE 2. The 272¼ before feet in the table is not a part of the scale.

1. Reduce 3sq. m. 325a. 2r. 37sq. rd. to square rods.

2. In 359317 square rods how many square miles, acres, roods, and rods?

3. Reduce 30sq. yd. 1sq. ft. 127sq. in. to square inches.

4. Reduce 39151sq. in. to higher denominations.

5. How many square feet in Fig. 1?

6. How many feet round Fig. 1?

7. Suppose each side of Fig. 2 to be 7 rods, what is the distance round it? How many square rods does it contain?

8. How many square rods in a rectangular field that is 17 rods wide and 35 rods long? How many acres? How many rods round this field?

9. A board containing 45 square feet is 15 feet long; how wide is it?

10. A flower-garden containing 288 square feet is 12 feet wide; how long is it? Ans. 24.

11. How many square yards of carpeting will be required to carpet a room that is 18 feet long and 15 feet wide?

12. At $2 per yard for carpeting that is a yard wide, what will be the cost of carpeting a room that is 5 yards square?

CUBIC OR SOLID MEASURE.

110. A SOLID OR BODY is any thing which has length, breadth, and thickness.

---

110. What is a Solid or Body?

111. A solid or body bounded by 6 rectangular faces, as Fig. 3, is called a RECTANGULAR PRISM.

Fig 3.

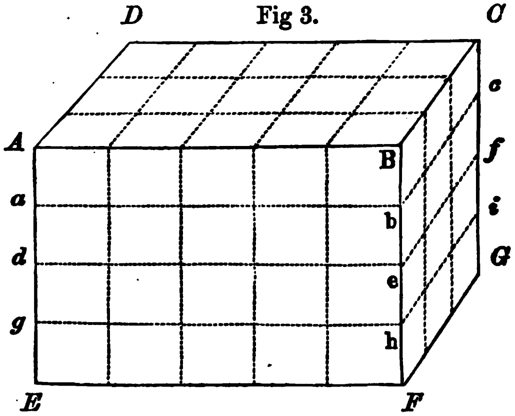
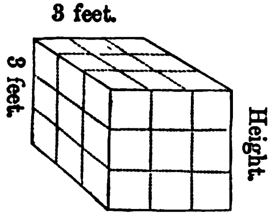


Fig. 4.



112. A rectangular prism bounded by six *square* faces, as Fig. 4, is called a CUBE.

113. To find the volume or solid contents of a rectangular prism, as Fig. 3, first find the area of the top face, *A B C D*, as in Art. 107; then going from *A*, *B*, and *C* downward 1 foot to *a*, *b*, and *c*, and passing a plane through *a*, *b*, and *c*, we shall cut off 15 solid feet, that is 5 × 3 × 1 solid

---

111. A Rectangular Prism? **112.** A Cube? **113.** How is the volume of a rectangular prism found?

feet. So, if a plane be passed through *d*, *e*, and *f*, it will cut off 30, or 5 × 3 × 2 solid feet, etc.; that is,

The continued product of the numbers representing the length, breadth, and height will give the volume or solid contents of a rectangular prism; thus, in Fig. 3, 5 × 3 × 4 = 60, (solid feet,) the *volume or contents.*

114. So, reversing the process in Art. 113

The volume divided by the area of the top face will give the height of the prism; the volume divided by the area of one end will give the length; and the volume divided by the area of one side will give the breadth or width; thus, in Fig. 3, 60 ÷ 15 = 4, the *depth;* 60 ÷ 12 = 5, the *length;* and 60 ÷ 20 = 3, the *breadth.*

115. SOLID OR CUBIC MEASURE is used in measuring things which have length, breadth, and thickness.

TABLE.

1728	Cubic Inches (c. in.)	make	1	Cubic Foot,	cu. ft.
27	Cubic Feet	"	1	Cubic Yard,	c. yd.
16	Cubic Feet	"	1	Cord Foot,	c. ft.
8 128	Cord Feet or Cubic Feet	"	1	Cord,	c.

c. yd.		cu. ft.		c. in.
		1	=	1728
1	=	27	=	46656

SCALE. Descending, 27, 1728; Ascending, 1728, 27.

NOTE 1. The numbers after 27, in this table, do not belong to the scale.

---

114. How the depth, when the volume and area of the top face are known? How the length, when the volume and area of one end are known? How the breadth, when the volume and area of one side are known? **115.** For what is Solid Measure used? Table? Scale? Note 1?

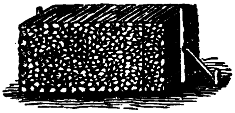


Fig. 5.

NOTE 2. A pile of wood, Fig. 5, that is 8 feet long, 4 feet wide, and 4 feet high, measures a cord, and one foot in length of such a pile is a cord foot.

NOTE 3. A *Perch* of building stone or masonry contains 24¾ cubic feet. A pile 16½ feet long, 1½ feet wide, and 1 foot high measures a perch.

NOTE 4. Transportation companies often estimate freight, especially of light articles, by the space occupied, rather than by the actual weight. In this estimate, from 25 or 30 to 150 or 175 cubic feet are called a ton. This is called *arbitrary* weight, and it varies with different transportation companies.

1. How many cubic inches in 33c. yd. 24cu. ft. 1635c. in.?

2. Reduce 1582755c. in. to higher denominations.

3. Reduce 15c. yd. 18cu. ft. 1727c. in. to cubic inches.

4. In 5 c. 6c. feet, 9 cubic feet, 125 c. in. how many cubic inches?

5. If 40 cu. ft. make one ton, how many tons, cubic feet, etc., in 347859 cubic inches?

6. How many cubic feet are there in Fig. 3? How many square feet in the top face of Fig. 3? How many in the front side? How many in the right-hand end? How many in the whole surface of Fig. 3.

7. How many cubic feet in a cubical box whose edges are 2 feet in length? How many cubic inches? How many square feet in its surface?

8. How many cords of wood in a pile that is 32 ft. long, 4 ft. wide, and 6 ft. high? How many cord feet? Cubic feet? Cubic inches?

9. A rectangular block of marble which contains 88 cubic feet, is 11 feet long and 4 feet wide; how thick is it?

---

115. Note 2? Note 3? Note 4?

10. A grain-bin which holds 30 cubic feet of grain is 3 feet deep and 2 feet wide; how long is it? Ans., 5 feet.

11. My cistern is 18 feet long, 15 feet wide, and 10 feet deep. By a pipe 6 cubic feet of water enter every minute; in how many minutes will the cistern be filled?

LIQUID MEASURE.

116. Liquid Measure is used in measuring all liquids.

TABLE.

4 Gills (gi.)	make	1 Pint,	pt.
2 Pints	"	1 Quart,	qt.
4 Quarts	"	1 Gallon,	gal.

				pt.		gi.
		qt.		1	=	4
gal.		1	=	2	=	8
1	=	4	=	8	=	32

Scale. Descending, 4, 2, 4; Ascending, 4, 2, 4.

Note 1. The United States Standard Unit of Liquid Measure is the old English wine gallon, which contains 231 cubic inches.

Note 2. It has been customary to measure milk, and also beer, ale and other malt liquors, by beer measure, the gallon containing 282 cubic inches, but this custom is fast going out of use.

Note 3. Casks of various capacities, from 50 to 150 or more gallons, are indiscriminately called hogsheads, pipes, butts, tuns, etc. Those containing from 30 to 40 gallons are called barrels.

1. Reduce 9gal. 3qt. 1pt. 2gi. to gills.
2. Reduce 318 gills to pints, quarts, etc.
3. Reduce 12gal. 1pt. to gills.
4. Reduce 573 gills to higher denominations.

---

116. For what is Liquid Measure used? Table? Scale? Note 1?

5. How many bottles, each containing 3qt. 1pt. 2gi., can be filled from a cask which contains 46gal. 3qt. 1pt.?

6. How many gallons of molasses in 24 jugs, each containing 1gal. 2qt. 1pt.?

DRY MEASURE.

117. Dry Measure is used in measuring grain, fruit, potatoes, salt, charcoal, etc.

TABLE.

2 pts. (pt.)	make	1 Quart,	qt.
8 Quarts	"	1 Peck,	pk.
4 Pecks	"	1 Bushel,	bush.

				qt.		pt.
		pk.		1	=	2
bush.		1	=	8	=	16
1	=	4	=	32	=	64

Scale. Descending, 4, 8, 2; Ascending, 2, 8, 4.

Note. The bushel measure is 18½ inches in diameter and 8 inches deep, and contains a little less than 2150⅖ solid inches, or nearly 9⅓ wine gallons. Consequently 4 quarts or half a peck of oats should measure nearly 38 cubic inches more than a gallon of wine; and a quart of berries, or any other article measured by Dry Measure, should contain nearly 9½ cubic inches more than a quart of wine or any other liquid.

1. Reduce 3bush. 2pk. 7qt. 1pt. to pints.
2. Reduce 239 pints to quarts, pecks, etc.
3. How many pints in 25bush. 1pk. 5qts. 1pt.?
4. How many pints in 17bush. 3qt.?

---

116. Note 2? Note 3? **117.** For what is Dry Measure used? Table? Scale? What are the dimensions of the bushel measure? How many cubic inches does it contain? How many wine gallons? How much ought a quart of berries to exceed a quart of milk?

5. Reduce 759 pints to higher denominations.

6. Reduce 8573 pints to higher denominations.

7. What is the cost of 2bush. 3pk. of grass seed, at $2 a peck?

TIME.

118. Time is used in measuring duration. The natural divisions of time are days, months, (moons), seasons, and years. The artificial divisions are seconds, minutes, hours, weeks, etc.

TABLE.

60 Seconds (sec.)	make	1 Minute,	m.
60 Minutes	"	1 Hour,	h.
24 Hours	"	1 Day,	d.
7 Days	"	1 Week,	wk.
4 Weeks	"	1 Lunar Month,	l. m.
13 Months, 1 Day, and 6 Hours	"	1 Julian Year,	J. yr.
12 Calendar Months (=365 or 366 Days),		1 Civil Year,	c. yr.
100 Years	make	1 Century,	C.

					m.	sec.
				h.	1 =	60
			d.	1 =	60 =	3600
		wk.	1 =	24 =	1440 =	86400
	l. m.	1 =	7 =	168 =	10080 =	604800
J. yr.	1 =	4 =	28 =	672 =	40320 =	2419200
1 =	$13\frac{5}{112}$ =	$52\frac{5}{28}$ =	$365\frac{1}{4}$ =	8766 =	525960 =	31557600

SCALE. Descending, 4, 7, 24, 60, 60; Ascending, 60, 60, 24, 7, 4.

---

118. For what is Time used? What are its natural divisions? Artificial divisions? Table?

Note 1. The names of the seasons and of the calendar months and the number of days in the several months, are as follows: —

Seasons.	Months.	No. of Days.
Winter.	1st. JANUARY	31
	2d. FEBRUARY	28, in leap year 29
Spring.	3d. MARCH	31
	4th. APRIL	30
	5th. MAY	31
Summer.	6th. JUNE	30
	7th. JULY	31
	8th. AUGUST	31
Autumn.	9th. SEPTEMBER	30
	10th. OCTOBER	31
	11th. NOVEMBER	30
Winter.	12th. DECEMBER	31

Note 2. The number of days in each month may be easily remembered by committing the following lines: —

Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Save the second month alone,
Which has just eight and a score
Till leap year gives it one more.

Note 3. A solar year, that is, a year by the sun, is very nearly 365 days, 5 hours, 48 minutes and 50 seconds.

1. How many seconds in 18h. 27m. 30sec.?
2. Reduce 12850 seconds to higher denominations.
3. Reduce 4d. 22h. 57m. 54sec. to seconds.
4. Reduce 9wk. 15h. 19sec. to seconds.
5. Reduce 452897 seconds to higher denominations.
6. In 7 centuries how many calendar months?
7. Reduce 10800 calendar months to centuries.

CIRCULAR MEASURE.

119. Circular Measure is used in surveying, navigation, geography, astronomy, etc., for measuring angles, determining latitude, longitude, etc.

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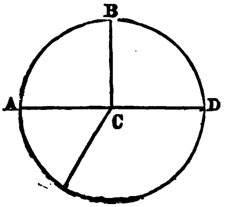
118. Scale? What are the names of the calendar months? How many days in each? In what season is each? The number of each from the beginning of the year? Length of a solar year?

TABLE.

60 Seconds (60″)	make	1 Minute,	1′
60 Minutes	"	1 Degree,	1°
30 Degrees	"	1 Sign,	s.
12 Signs, or 360°	"	1 Circumference,	circ.

			1′ =	60″
	s.	1° =	60 =	3600
circ.	1 =	30 =	1800 =	108000
1 =	12 =	360 =	21600 =	1296000

Scale. Descending, 12, 30, 60, 60; Ascending, 60, 60, 30, 12.



Note. A curved line is a figure bounded by a curved line, all parts of the curve being equally distant from the center of the circle.

The *Circumference* is the curve which bounds the circle. An *Arc* is any portion of the circumference, as A B or B D. An arc equal to a quarter of the circumference, or 90°, is called a *quadrant.* A *Radius* is a line drawn from the center to the circumference, as C A or C B. A *Diameter* is a line drawn *through* the center and limited by the curve, as A D.

1. In 15° 38′ 29″ how many seconds?
2. Reduce 78695″ to degrees, etc.
3. Reduce 2° 27′ 39″ to seconds.
4. In 5s. 17° how many minutes?
5. Reduce 276892″ to signs, etc.
6. Reduce 17s. 21° 28′ 3″ to seconds.

---

119. For what is Circular Measure used? Table? Scale? What is a Circle? Circumference? Arc? Quadrant? Radius? Diameter?

MISCELLANEOUS EXAMPLES IN REDUCTION.

1. In 7£ 15s. 6d. 3qr. how many farthings?
2. Reduce 67219qr. to pounds sterling, etc.
3. Reduce 10oz. 17dwt. 15gr. to grains.
4. Change 27619gr. to pounds, etc.
5. In 7oz. 5dr. 2sc. 12gr. of opium, how many grains?
6. Reduce 17 tons 16cwt. 3qr. to quarters.
7. Change 627243oz. to tons, etc.
8. In 7yd. 3qr. 2na. 1in. how many inches?
9. Reduce 742 inches to yards. etc.
10. Change 5fur. 13rd. 7ft. 10in. to inches.
11. Reduce 273894 inches to miles, etc.
12. In 27 fathoms, how many inches?
13. John Smith's horse is 15 hands high; how many inches high is he?
14. In 7m. 3fur. 7ch. 2rd. how many links?
15. Reduce 3a. 2r. 27sq. rd. 127sq. ft. 126sq. in. to square inches.
16. How many cu. in. in 17 cords?
17. Reduce 76493c. in. to cords, etc.
18. How many gills in 27gal. 3qt. 1pt. 3gi.?
19. Reduce 643gi. to gallons, etc.
20. Change 46bu. 3pk. 6qt. 1pt. to pints.
21. In 874qt. how many bushels?
22. Reduce 17h. 56m. 43sec. to seconds.
23. Reduce 178cwt. 2qr. 10lb. to ounces.
24. Reduce 10yd. 2na. to nails.
25. Reduce 726890 inches to miles.

DEFINITIONS AND GENERAL PRINCIPLES.

120. All numbers are *even* or *odd.*

An Even Number is a number that is divisible by 2; as 2, 4, 8, 12.

An Odd Number is a number that is *not* divisible by 2; as 1, 3, 5, 11, 19.

121. All numbers are *prime* or *composite.*

A Prime Number is a number that is divisible by no whole number except *itself* and *one;* as 1, 2, 3, 5, 7, 11, 19.

A Composite Number is a number that is divisible by other numbers besides itself and 1; thus, 6 is composite, because it is divisible by 2 and by 3; 12 is composite, because it is divisible by 2, 3, 4, and 6; 25 is composite, because it is divisible by 5 and 5.

Factoring Numbers.

122. The Factors of a number are those numbers whose continued product is the number; thus, 3 and 7 are the factors of 21; 3 and 6, or 3, 3, and 2 are the factors of 18; etc.

The *prime* factors of a number are those *prime* numbers whose continued product is the number; thus, the prime factors of 12 are 2, 2, and 3; the prime factors of 36 are 2, 2, 3, and 3; etc.

Note. Since 1, as a factor is useless, it is not here enumerated.

---

120. What is an Even Number? An Odd Number? **121.** A Prime Number? What is the only *even* prime number? What is a Composite Number?

122. What are the Factors of a number? What are the *prime* factors of a number?

TABLE OF PRIME NUMBERS FROM 1 TO 997.

1	41	101	167	239	313	397	467	569	643	733	823	911
2	43	103	173	241	317	401	479	571	647	739	827	919
3	47	107	179	251	331	409	487	577	653	743	829	929
5	53	109	181	257	337	419	491	587	659	751	839	937
7	59	113	191	263	347	421	499	593	661	757	853	941
11	61	127	193	269	349	431	503	599	673	761	857	947
13	67	131	197	271	353	433	509	601	677	769	859	953
17	71	137	199	277	359	439	521	607	683	773	863	967
19	73	139	211	281	367	443	523	613	691	787	877	971
23	79	149	223	283	373	449	541	617	701	797	881	977
29	83	151	227	293	379	457	547	619	709	809	883	983
31	89	157	229	307	383	461	557	631	719	811	887	991
37	97	163	233	311	389	463	563	641	727	821	907	997

123. To resolve or separate a number into its prime factors we have the following:

RULE. *Divide the given number by any prime number greater than one, that will divide it; divide the* QUOTIENT *by any prime number greater than one that will divide* IT, *and so on till the quotient is prime. The several divisors and last quotient will be the prime factors sought.*

1. **What are the prime factors of 5768?**

OPERATION.

2) 5 7 6 8

2) 2 8 8 4

2) 1 4 4 2

7) 7 2 1

1 0 3 Ans. 2, 2, 2, 7, 1 0 3.

---

123. Rule for resolving a number into its prime factors?

2. Resolve 680 into its prime factors.

Ans. 2, 2, 2, 5, and 17.

3. Resolve 846 into its primo factors.

Ans. 2, 3, 3, 47.

4. What are the prime factors of 200 ?

Ans. 2, 2, 2, 5, 5.

5. Resolve 984 into its prime factors.

Ans. 2, 2, 2, 3, 41.

GREATEST COMMON DIVISOR.

124. A COMMON DIVISOR of two or more numbers is *any number that will divide each of them without remainder ;* thus 3 is a common divisor of 12, 18, and 30.

125. The GREATEST COMMON DIVISOR of two or more numbers is the *greatest* number that will divide each of them without remainder; thus, 6 is the greatest common divisor of 12, 18, and 30.

NOTE. A *divisor* of a number is often called *a measure* of the number, also an *aliquot part* of the number.

126. To find the greatest common divisor we have the following:

RULE 1. *Divide the greater of two numbers by the less, and, if there be a remainder, divide the divisor by the remainder, and continue dividing the last divisor by the last remainder until nothing remains ; the last divisor is the greatest common divisor of the two numbers. Cr,*

RULE 2. *If more than two numbers are given, find the greatest divisor of two of them, then of this divisor and a third number, and so on until all the numbers have been taken ; the last divisor will be the divisor sought.*

---

124. What is a common divisor? **125.** What is the greatest common divisor?

126. Rule for finding the greatest common divisor of two numbers? Second rule for finding greatest common divisor of more than two numbers?

1. What is the greatest common divisor of 16 and 44?

OPERATION.

16) 44 (2
32
—
12) 16 (1
12
—
Ans., 4) 12 (3
12
—

2. What is the greatest common divisor of 8, 12, 28?

OPERATION.

8) 12 (1
8
—
4) 8 (2
8
—
Ans., 4) 28 (7
28
—

3. Find the greatest common divisor of 9, 12, 18 and 24. Ans. 3.

4. What is the greatest common divisor of 24, 40, 68? Ans. 4.

5. What is the greatest common divisor of 144, 17, and 1728?

6. What is the greatest common divisor of 72, 45, 999?

7. What is the greatest common divisor of 1825, 640, 50?

LEAST COMMON MULTIPLE.

127. A MULTIPLE of a number is any number which is *divisible* by that number; thus, 15 is a multiple of 5 and also of 3; 21 is a multiple of 7 and of 3.

128. A COMMON MULTIPLE of two or more numbers, is any number which is divisible by each of the given numbers; thus, 48 is a common multiple of 4, 6, and 8.

129. The LEAST COMMON MULTIPLE of two or more numbers, is the *least* number that is divisible by each of the given numbers; thus, 24 is the least common multiple of 4, 6, and 8.

---

127. What is a Multiple of a number?
128. A Common Multiple of two or more numbers?
129. The Least Common Multiple?

130. To find the least common multiple of two or more numbers:

RULE. *Having set the given numbers in a line, divide by any* PRIME *number that will divide two or more of them, and set the quotients and undivided numbers in a line beneath; proceed with this line as with the first, and so continue until no two of the numbers can be divided by any number greater than one; the continued product of the divisors and numbers in the last line will be the multiple sought.*

Ex. 1. What is the least common multiple of 6, 8, 12, 16, 18, 24?

OPERATION.

$$\begin{array}{r|rrrrrr}
2 & 6, & 8, & 12, & 16, & 18, & 24 \\ \hline
2 & 3, & 4, & 6, & 8, & 9, & 12 \\ \hline
2 & 3, & 2, & 3, & 4, & 9, & 6 \\ \hline
3 & 3, & 1, & 3, & 2, & 9, & 3 \\ \hline
 & 1, & 1, & 1, & 2, & 3, & 1
\end{array}$$

$2 \times 2 \times 2 \times 3 \times 2 \times 3 = 144$, Ans.

2. Find the least common multiple of 5, 10, 12, 15, 20, 24. 120, Ans.

3. Find the least common multiple of 7, 8, 12, 14, 16, 21. Ans. 336.

4. Find the least common multiple of 24, 72, 18, 48.

5. Find the least common multiple of 10, 15, 24, 18, 32.

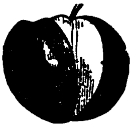
6. Find the least common multiple of 21, 7, 36, 42, 84, 75.

---

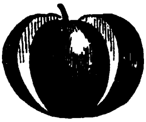
130. Rule for finding the Least Common Multiple?

COMMON FRACTIONS.

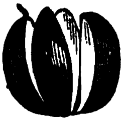
131. If a single thing (an apple, for instance,) is divided into two equal parts, one of these parts is called one half, (written $\frac{1}{2}$) ; if divided into three equal parts, one of these parts is called one third ($\frac{1}{3}$).



Halves.

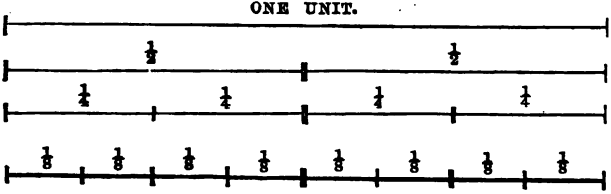


Thirds.



Fourths.

And so, if we divide a unit or single thing into four, five, six, etc. equal parts, one of these parts is called one fourth ($\frac{1}{4}$), one fifth ($\frac{1}{5}$), one sixth ($\frac{1}{6}$), etc.



132. A FRACTION is an expression representing one or more of the equal parts of a unit.

133. A COMMON OR VULGAR FRACTION is expressed by two numbers, one above and the other below a line; thus $\frac{1}{2}$ (one half), $\frac{2}{5}$ (two fifths), &c.

(a) The number above the line is called the Numerator, and the number below the line is called the Denominator.

(b) The Denominator shows into how many parts the unit is divided, and gives the name to the fraction.

---

132. What is a Fraction? **133.** A Common Fraction? (a) Where do we write the numerator? Denominator? (b) What does the denominator show? (c) What the numerator? (d) What are both called?

(c) The Numerator shows how many of those parts are taken or expressed by the fraction.

(d) The numerator and denominator are called the terms of the fraction.

Write the following fractions: three fourths, two thirds, seven eighths, nine tenths, seven elevenths, eight fifteenths.

Read the following fractions: $\frac{7}{9}$, $\frac{4}{5}$, $\frac{3}{8}$, $\frac{6}{7}$, $\frac{5}{9}$, $\frac{9}{11}$, $\frac{12}{19}$, $\frac{17}{29}$.

134. A SIMPLE FRACTION has but one numerator and one denominator; as $\frac{3}{4}$, $\frac{6}{7}$, $\frac{13}{4}$.

135. A COMPOUND FRACTION is a fraction of a fraction; as $\frac{2}{3}$ of $\frac{6}{7}$, $\frac{5}{8}$ of $\frac{9}{10}$.

136. A PROPER FRACTION is one whose numerator is *less* than the denominator; as $\frac{5}{8}$, $\frac{3}{5}$, $\frac{2}{3}$.

137. An IMPROPER FRACTION is one whose numerator equals or exceeds the denominator; as $\frac{4}{4}$, $\frac{7}{7}$, $\frac{5}{2}$, $\frac{19}{7}$.

138. A MIXED NUMBER is a whole number and a fraction united; as, $7\frac{1}{2}$, $5\frac{3}{4}$, $27\frac{4}{7}$.

139. The terms of a fraction sustain to each other the relation of dividend and divisor, the numerator answering to the dividend and the denominator to the divisor.

That is, a fraction may be regarded as an expression of division. Hence,

The VALUE of a fraction is the QUOTIENT of the numerator divided by the denominator, as $\frac{9}{3} = 9 \div 3 = 3$.

It follows from this that the GENERAL PRINCIPLES of DIVISION (Arts. 85, 86, and 87) apply to all fractions.

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134. What is a Simple Fraction? **135.** Compound? **136.** Proper? **137.** Improper? **138.** What is a mixed number?

139. What relation do the terms of a fraction sustain to each other? Which term answers to the dividend? Which to the divisor? How may a fraction be regarded? To what is the value of a fraction equivalent? What principles before stated apply to fractions?

1. *Multiplying the numerator, if the denominator remains unaltered, multiplies the value of the fraction by the same number, as* $\frac{2}{4}\times 2=\frac{4}{4}$.

2. *Dividing the numerator, if the denominator remains unaltered, divides the value of the fraction by the same number, as* $\frac{2}{4}\div 2=\frac{1}{4}$.

In the above cases it will be seen that the *size* of the parts, (fourths,) remains the same, but the *number* of the parts is increased or diminished.

3. *Multiplying the denominator, if the numerator remains unaltered, divides the value of the fraction by the same number, as* $\frac{2}{4}\times 2=\frac{2}{8}$.

4. *Dividing the denominator, if the numerator remains unaltered, multiplies the value of the fraction by the same number, as* $\frac{2}{4}\div 2=\frac{2}{2}$.

In the last two cases it will be seen that the *number* of parts (numerators) remains the same, but the *size* of the parts (denominators) is increased or diminished.

5. *If the numerator and denominator are both multiplied or divided by the same number the value of the fraction is not altered, as* $\frac{2\times 2}{4\times 2}=\frac{4}{8}$ *or* $\frac{2\div 2}{4\div 2}=\frac{1}{2}$.

Hence, the following general law in regard to Fractions may be stated,

That any change in the NUMERATOR *causes a* LIKE *change in the value of the fraction; and any change in the* DENOMINATOR *causes an* OPPOSITE *change in the value of the fraction.*

Upon these principles all the following operations upon fractions depend.

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139. Give the 1st principle and illustrate it. The 2d principle. The 3d principle. The 4th principle. The 5th principle. What general law is given?

Case 1.

140. To reduce a mixed number to an improper fraction.

Ex. 1. In $7\frac{3}{5}$ how many fifths?

OPERATION.
$7\frac{3}{5}$
5
—
$\frac{38}{5}$ Ans.

In a unit there are five fifths; and in seven units there are seven times five fifths, or 35 fifths, which with the 3 fifths in the example = 38 fifths = $\frac{38}{5}$.

Rule. *Multiply the whole number by the denominator of the fraction; to the product add the numerator, and under the sum write the denominator.*

2. Reduce $17\frac{5}{9}$ to an improper fraction. Ans. $\frac{158}{9}$.
3. Reduce $26\frac{11}{13}$ to an improper fraction. Ans. $\frac{349}{13}$.
4. Reduce $43\frac{6}{7}$ to an improper fraction. Ans. $\frac{307}{7}$.
5. Reduce $56\frac{2}{3}$ to an improper fraction. Ans. $\frac{170}{3}$.
6. Reduce $85\frac{7}{12}$ to an improper fraction. Ans. $\frac{1027}{12}$.
7. In $19\frac{5}{14}$ how many fourteenths?
8. How many seventeenths in $38\frac{11}{17}$? Ans. $\frac{657}{17}$.
9. Reduce $49\frac{17}{23}$ to an improper fraction.

Note. To reduce a whole number to a fraction having any given denominator, multiply the whole number by the proposed denominator, and under the product write the denominator.

Case 2.

141. To reduce an improper fraction to a whole or mixed number.

Ex. 1. How many units in $\frac{17}{4}$?

$\frac{17}{4} = 17 \div 4 = 4\frac{1}{4}$

In one unit there are four fourths, and in seventeen fourths there are as many units as four is contained times in seventeen.

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140. Explain the operation in Case 1. Rule for reducing a mixed number to an improper fraction?

141. Rule for reducing an improper fraction to a whole or mixed number?

Rule. *Divide the numerator by the denominator; if there is any remainder, place it over the divisor, and annex the fraction so formed to the quotient.*

2. Reduce $\frac{19}{7}$ to a mixed number. Ans, $2\frac{5}{7}$.
3. Reduce $\frac{29}{5}$ to a mixed number. Ans. $5\frac{4}{5}$.
4. Reduce $\frac{39}{11}$ to a mixed number. Ans. $3\frac{6}{11}$.
5. Reduce $\frac{87}{19}$ to a mixed number.
6. Reduce $\frac{267}{28}$ to a mixed number. Ans. $9\frac{15}{28}$.
7. Reduce $\frac{439}{42}$ to a mixed number. Ans. $10\frac{19}{42}$.
8. Reduce $\frac{691}{13}$ to a mixed number.
9. Reduce $\frac{84}{12}$ to a whole number. Ans. 7.
10. Reduce $\frac{493}{29}$ to a whole number.

Note. The denominator of a fraction being a divisor, it follows that whenever the denominator exactly measures the numerator, the quotient will be a whole number. (See Exs. 9 and 10.)

Case 3.

142. To reduce a fraction to its lowest terms.

Ex. 1. Reduce $\frac{24}{36}$ to its lowest terms.

1st operation

$\begin{matrix}2)\\2)\end{matrix}\frac{24}{36}=\frac{12}{18}$

$\begin{matrix}2)\\2)\end{matrix}\frac{12}{18}=\frac{6}{9}$

$\begin{matrix}3)\\3)\end{matrix}\frac{6}{9}=\frac{2}{3}$ Ans.

Rule 1. *Divide each term by any factor common to them, then divide these quotients by any factor common to* THEM, *and so proceed till the quotients are mutually prime.* (Art. 139, 5th.)

2d operation. Find the greatest common divisor, (Art. 125,) and by it divide both terms of the fraction.

$\begin{matrix}12)\\12)\end{matrix}\frac{24}{36}=\frac{2}{3}$ Ans.

Rule 2. *Divide each term by their greatest common divisor.*

2. Reduce $\frac{18}{30}$ to its lowest terms. Ans. $\frac{3}{5}$.
3. Reduce $\frac{56}{64}$ to its lowest terms. Ans. $\frac{7}{8}$.
4. Reduce $\frac{112}{144}$ to its lowest terms.

---

142. Rule for reducing a fraction to its lowest terms? Second rule for reducing a fraction to its lowest terms?

5. Reduce $\frac{72}{120}$ to its lowest terms. Ans. $\frac{3}{5}$.
6. Reduce $\frac{216}{288}$ to its lowest terms.
7. Reduce $\frac{32}{256}$ to its lowest terms. Ans. $\frac{1}{8}$.
8. Reduce $\frac{162}{486}$ to its lowest terms.

CASE 4.

143. To multiply a fraction by a whole number.

Ex. 1. Multiply $\frac{7}{8}$ by 4.

1st OPERATION.
$\frac{7}{8} \times 4 = \frac{28}{8} = \frac{7}{2}$

It is just as evident that 4 times 7 eighths ($\frac{7}{8}$) are 28 eighths ($\frac{28}{8}$) as it is that 4 times 7 boys are 28 boys.

2d OPERATION.
$\frac{7}{8} \div 4 = \frac{7}{2}$

If we divide the denominator by 4 we obtain the same result as before.

In the first operation we increase the *number* of the parts four-fold, and in the second, we increase the *size* or *value* of the parts four-fold while the number of parts remains the same. Hence the following

RULE 1 *Multiply the numerator by the whole number.* *Or,*

RULE 2. *Divide the denominator by the whole number.*

2. Multiply $\frac{2}{11}$ by 3. Ans. $\frac{6}{11}$.
3. Multiply $\frac{7}{16}$ by 8. Ans. $\frac{56}{16}$ or $\frac{7}{2}$.
4. Multiply $\frac{17}{29}$ by 5. Ans. $\frac{85}{29} = 2\frac{27}{29}$.
5. Multiply $\frac{6}{28}$ by 14. Ans. $\frac{84}{28}$ or $\frac{6}{2} = 3$.
6. Multiply $\frac{9}{42}$ by 3. Ans. $\frac{27}{42}$ or $\frac{9}{14}$.
7. Multiply $\frac{23}{864}$ by 15. Ans. $\frac{345}{864}$.
8. Multiply $\frac{83}{240}$ by 12.

---

143. First rule for multiplying a fraction by a whole number? Second rule?

9. Multiply $\frac{68}{425}$ by 21.
10. Multiply $\frac{54}{89}$ by 117. Ans. $\frac{6318}{89}$.
11. Multiply $\frac{47}{85}$ by 17.
12. Multiply $\frac{67}{246}$ by 34. Ans. $\frac{1139}{123}$.

(a) To multiply a mixed number by a whole number.

13. Multiply $4\frac{4}{5}$ by 8.

1st OPERATION.

$$4\frac{4}{5} = \frac{24}{5}$$
$$\frac{24}{5} \times 8 = \frac{192}{5} = 38\frac{2}{5} \text{ Ans.}$$

2d OPERATION.

$$\frac{4}{5} \times 8 = \frac{32}{5} = 6\frac{2}{5}$$
$$4 \times 8 = 32.$$
$$32 + 6\frac{2}{5} = 38\frac{2}{5}.$$

Hence the following

Rule. *Reduce the mixed number to an improper fraction,* (*Art.* 140,) *and then multiply. Or, multiply the fraction and whole number separately and add the products together.*

14. Multiply $6\frac{5}{7}$ by 9. Ans. $60\frac{3}{7}$.
15. Multiply $7\frac{4}{13}$ by 26. Ans. 190.
16. Multiply $28\frac{6}{25}$ by 42. Ans. $1186\frac{2}{25}$.
17 Multiply $46\frac{9}{11}$ by 39.
18. Multiply $89\frac{7}{19}$ by 68.
19. Multiply $246\frac{11}{27}$ by 142.
20. Multiply $392\frac{4}{17}$ by 257.
21. Multiply $150\frac{1}{2}$ by 27.

Note. To multiply a whole number by a fraction is just the same as multiplying a fraction by a whole number. e. g. $\frac{6}{7} \times 4 = 4 \times \frac{6}{7}$.

Case 5.

144. To divide a fraction by a whole number.

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143. How is a mixed number multiplied by a whole number? Another method?

Ex. 1. Divide $\frac{6}{7}$ by 3. Ans. $\frac{2}{7}$.

1st OPERATION.
$\frac{6}{7} \div 3 = \frac{2}{7}$

One third of 6 apples is 2 apples; it is equally clear that one third of 6 sevenths ($\frac{6}{7}$) is 2 sevenths ($\frac{2}{7}$.)

2d OPERATION.
$\frac{6}{7} \times 3 = \frac{6}{21}$

If I divide $\frac{6}{7}$ by 1, the quotient will be $\frac{6}{7}$. Now if I divide it by 3 instead of 1, I obtain a quotient only one third as great, or $\frac{1}{3}$ of $\frac{6}{7} = \frac{6}{21}$. In this instance the *number* of parts remains the same, while the *size* of the parts is diminished.

RULE 1. *Divide the numerator by the whole number. Or,*
RULE 2. *Multiply the denominator by the whole number.* (Art. 139, 2nd and 3rd.)

2. Divide $\frac{16}{25}$ by 8. Ans. $\frac{2}{25}$.
3. Divide $\frac{18}{23}$ by 6. Ans. $\frac{3}{23}$.
4. Divide $\frac{17}{18}$ by 5. Ans. $\frac{17}{90}$.
5. Divide $\frac{13}{15}$ by 12. Ans. $\frac{13}{180}$.
6. Divide $\frac{26}{43}$ by 13. Ans. $\frac{2}{43}$.
7. Divide $\frac{35}{42}$ by 7. Ans. $\frac{5}{42}$.
8. Divide $\frac{56}{94}$ by 14.
9. Divide $\frac{146}{289}$ by 35.
10. Divide $\frac{267}{280}$ by 42.

NOTE. If the dividend be a mixed number first reduce it to an improper fraction, or divide the whole number and fraction separately and add the results.

11. Divide $26\frac{3}{4}$ by 6.
12. Divide $16\frac{3}{8}$ by 7. Ans. $2\frac{19}{56}$.
13. Divide $28\frac{7}{8}$ by 7.

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144. First rule for dividing a fraction by a whole number? Second rule? A mixed number how divided by a whole number?

14. Divide $69\frac{7}{8}$ by 13. Ans. $5\frac{3}{8}$.

15. Divide $211\frac{1}{5}$ by 12. Ans. $17\frac{3}{5}$.

CASE 6.

145. To multiply a fraction by a fraction.

Ex. 1. Multiply $\frac{3}{4}$ by $\frac{7}{8}$. Ans. $\frac{21}{32}$.

We first multiply the fraction $\frac{3}{4}$ by 7, (Art. 143, Rule 1.) and obtain $\frac{21}{4}$. Now, as 7 is eight times the true multiplier $\frac{7}{8}$, the product is 8 times too large; and we obtain the true product by dividing $\frac{21}{4}$ by 8 (Art. 144, Rule 2.) $\frac{3}{4} \times 7 = \frac{21}{4}$, and $\frac{21}{4} \div 8 = \frac{21}{32}$. Hence,

RULE. *Multiply the numerators together for a new numerator, and the denominators for a new denominator.*

2. Multiply $\frac{\overset{2}{\cancel{6}}}{7} \times \frac{2}{\cancel{3}}$ Ans. $\frac{4}{7}$.

In the above example, we have the factor 3 in the numerator of the $\frac{6}{7}$, and also in the denominator of the $\frac{2}{3}$. These we reject in the operation, since this is equivalent to dividing both terms of the product by 3, which (Art. 139, 5th) does not alter the value of the fraction, and obtain the answer in its lowest terms. This process of cancellation may be employed advantageously in many cases, as the principle is the same, as when applied in division. (See Art. 88.)

3. Multiply $\frac{17}{\underset{7}{\cancel{35}}} \times \frac{\cancel{5}}{8} = \frac{17}{56}$, Ans.

4. Multiply $\frac{5}{13}$ by by $\frac{19}{20}$. Ans. $\frac{19}{52}$.

5. Multiply $\frac{28}{43}$ by $\frac{14}{15}$. Ans. $\frac{392}{645}$.

6. Multiply $\frac{42}{59}$ by $\frac{17}{28}$. Ans. $\frac{51}{118}$.

7. Multiply $\frac{87}{56}$ by $\frac{42}{75}$.

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145. Rule for multiplying a fraction by a fraction?

Though the above rule will give a common denominator, yet it will not always give the *least* common denominator.

Ex. 7. Reduce $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{12}$, $\frac{13}{16}$ to equivalent fractions having the least common denominator.

OPERATION.

$$\begin{array}{r|rrrr} 4) & 4, & 8, & 12, & 16 \\ \hline 2) & 1, & 2, & 3, & 4 \\ \hline & 1, & 1, & 3, & 2 \end{array}$$

$4 \times 2 \times 3 \times 2 = 48 =$ L. C. M. $=$ Least Com. Denom. (Art. 130, Rule.)

$48 \div 4 = 12$, and $12 \times 3 = 36 =$ 1st numerator.
$48 \div 8 = 6$, and $6 \times 5 = 30 =$ 2d numerator.
$48 \div 12 = 4$, and $4 \times 7 = 28 =$ 3d numerator.
$48 \div 16 = 3$, and $3 \times 13 = 39 =$ 4th numerator.
Ans., $\frac{36}{48}$, $\frac{30}{48}$, $\frac{28}{48}$, and $\frac{39}{48}$, the several equivalent fractions.

EXPLANATION. We first find the least common multiple of the denominators 4, 8, 12, and 16, which is the least common denominator. We then divide this denominator by each of the denominators of the given fractions, and multiply each quotient by its corresponding numerator. Hence we have the

RULE. *Reduce all the fractions (if necessary) to their lowest terms. Find the least common multiple of all the denominators for a common denominator. Divide this multiple by each of the given denominators, and multiply the several quotients by their respective numerators for new numerators.*

NOTE. It will be seen that both the above rules are based upon the principle Art. 139, 5th.

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147. Rule for obtaining the least common denominator of several fractions? On what principle does this rule and the former one depend? Explain.

8. Reduce $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{9}$, $\frac{11}{12}$ to equivalent fractions having the least common denominator. Ans. $\frac{24}{36}$, $\frac{30}{36}$, $\frac{28}{36}$, $\frac{33}{36}$.

9. Reduce $\frac{3}{7}$, $\frac{7}{8}$, $\frac{13}{14}$ to equivalent fractions having the the least common denominator.

10. Reduce $\frac{5}{9}$, $\frac{4}{5}$, $\frac{7}{10}$, $\frac{1}{3}$ to equivalent fractions having the least common denominator.

CASE 9.

148. Numbers that are of the same kind can be added together. For example, 6 pens + 7 pens + 5 pens = 18 pens. 2 hats + 5 hats = 7 hats; and for the same reason $\frac{3}{4} + \frac{2}{4} + \frac{1}{4} = \frac{6}{4}$. $\frac{5}{9} + \frac{2}{9}, + \frac{1}{9} = \frac{8}{9}$, etc. But numbers which are *not alike, cannot be added.* We cannot say 6 knives + 4 pens = 10 pens, or 10 knives. Neither can we say $\frac{3}{5}$ bush. + $\frac{4}{5}$ qt. = $\frac{7}{5}$ bush. or $\frac{7}{5}$ qt. NUMBERS MUST BE OF THE SAME KIND if we would ADD THEM TOGETHER. Hence,

To add fractions we have the following

RULE. *Reduce the fractions to equivalent fractions having a common denominator; after which, write the sum of the numerators over the common denominator.*

Ex. 1. Add $\frac{5}{18}$, $\frac{3}{18}$, and $\frac{7}{18}$ together. Ans. $\frac{15}{18}$.

2. Add $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{5}{7}$ together. Ans. $\frac{208}{105} = 1\frac{103}{105}$.

3. Add $\frac{3}{7}$, $\frac{5}{9}$, $\frac{5}{14}$ and $\frac{2}{3}$. Ans. $\frac{253}{126} = 2\frac{1}{126}$.

4. $\frac{5}{7} + \frac{4}{5} + \frac{7}{9} + \frac{5}{6} =$ what?

5. $\frac{3}{11} + \frac{3}{4} + \frac{2}{3} + \frac{7}{8} =$ what? Ans. $\frac{677}{264} = 2\frac{149}{264}$.

6. $\frac{7}{9} + \frac{2}{3} + \frac{3}{4} + \frac{7}{8} =$ what? Ans. $\frac{221}{72} = 3\frac{5}{72}$.

7. $\frac{3}{5}$ of $\frac{5}{7} + \frac{2}{3}$ of $\frac{7}{8} =$ what? Ans. $\frac{85}{84} = 1\frac{1}{84}$.

NOTE 1. Compound fractions must be first reduced to simple fractions.

8. $\frac{5}{8}$ of $\frac{9}{10} + \frac{6}{7}$ of $\frac{7}{9} =$ what? Ans. $\frac{59}{48} = 1\frac{11}{48}$

9. $\frac{3}{5}$ of $\frac{2}{3} + \frac{14}{15}$ of $\frac{5}{6} =$ what?

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148. Can numbers *not alike* be added? Rule for adding fractions?

10. $\frac{6}{7}$ of $\frac{3}{8}$ of $\frac{3}{4} + \frac{5}{9}$ of $\frac{7}{10} =$ what?

11. Add $4\frac{2}{3}$, $6\frac{7}{8}$, and $5\frac{3}{5}$ together.

$4 + 6 + 5 = 15$, $\frac{2}{3} + \frac{7}{8} + \frac{3}{5} = 2\frac{17}{120}$, and $15 + 2\frac{17}{120} = 17\frac{17}{120}$, Ans.

Note 2. Mixed numbers may be reduced to improper fractions; or the whole numbers and the fractions may be added separately, as above, and then their sums united.

12. Add $4\frac{1}{8}$ and $7\frac{4}{5}$ together. Ans. $11\frac{37}{40}$.

13. Add $5\frac{2}{7}$, $7\frac{4}{5}$, and $\frac{6}{7}$ together.

14. $16\frac{2}{3} + 14\frac{5}{9} + 18\frac{3}{7} =$ what?

Practical Questions in Addition of Fractions.

1. John bought a top for $\frac{1}{4}$ of a dollar, a knife for $\frac{7}{8}$ of a dollar, and a ball for $\frac{1}{8}$ of a dollar; how much did they cost him? Ans. \$$1\frac{1}{4}$.

2. Jane bought a work-basket for $\frac{7}{8}$ of a dollar, a pair of gloves for $1\frac{3}{5}$ dollars, and gave $\frac{1}{2}$ dollar to some poor persons; how much did she expend in all?

3. A lady bought several remnants of cloth. One piece was $\frac{3}{4}$ yd. long, another $\frac{7}{8}$ yd., and a third $\frac{1}{2}$ yd.; how much cloth did she buy in all? Ans. $2\frac{1}{8}$ yds.

4. A coal dealer sold coal to three men, as follows: To one $2\frac{3}{5}$ tons, to another $5\frac{2}{3}$ tons, to the third $6\frac{7}{8}$ tons; how much did he sell to all?

5. Three men bought a horse. One paid $51\frac{1}{4}$ dollars, another $67\frac{3}{5}$ dollars, and the third paid $76\frac{5}{8}$ dollars; how much did the horse cost?

Case 10.

149. The remarks under Case 9 apply with equal force to questions in *Subtraction*. We cannot take 4 pens from 6 knives; nor can we take $\frac{3}{4}$ of a gallon from $\frac{7}{8}$ of a pound. NUMBERS MUST BE OF THE SAME KIND or the SUBTRACTION cannot be performed. Hence

To subtract fractions we have the following

RULE. *Reduce the fractions to equivalent fractions having a common denominator, and then write the difference of the numerators over the common denominator.*

Ex. 1. From $\frac{7}{8}$ take $\frac{4}{8}$. Ans. $\frac{3}{8}$.
2. From $\frac{5}{7}$ take $\frac{3}{7}$. Ans. $\frac{2}{7}$.
3. From $\frac{5}{6}$ take $\frac{3}{5}$. Ans. $\frac{7}{30}$.
4. From $\frac{6}{7}$ take $\frac{3}{11}$. Ans. $\frac{45}{77}$.
5. From $\frac{9}{10}$ of $\frac{5}{12}$ take $\frac{3}{7}$ of $\frac{2}{9}$. Ans. $\frac{47}{168}$.
6. From $4\frac{2}{3}$ take $2\frac{5}{7}$. Ans. $\frac{41}{21} = 1\frac{20}{21}$.

NOTE 1. Whenever mixed numbers occur in the question they must be reduced to improper fractions.

7. Subtract $\frac{2}{7}$ from $1\frac{7}{8}$. Ans. $\frac{89}{56} = 1\frac{33}{56}$.
8. $4\frac{3}{8} - 2\frac{4}{7} =$ what?
9. $5\frac{6}{11} - 4\frac{8}{9} =$ what?
10. $6\frac{3}{4} - 5\frac{1}{3} =$ what? Ans. $1\frac{5}{12}$.
11. $\frac{3}{5}$ of $\frac{10}{11} - \frac{2}{7}$ of $\frac{2}{13} =$ what?
12. $\frac{5}{7}$ of $5\frac{2}{3} - \frac{5}{8}$ of $3\frac{1}{5} =$ what?

150. PRACTICAL QUESTIONS IN SUBTRACTION OF FRACTIONS.

1. A boy having $\frac{7}{8}$ of a qt. of nuts, gave away $\frac{1}{3}$ of a qt.; what part of a quart had he left? Ans. $\frac{13}{24}$.

2. A merchant having a piece of cloth containing $12\frac{3}{5}$ yds. sold $7\frac{5}{8}$ yds.; how much had he left? Ans. $4\frac{39}{40}$ yds.

3. A farmer had a field containing $23\frac{5}{8}$ acres. Of this, $6\frac{2}{3}$ acres were planted with corn, and the remainder bore grass; how much grass land was there in the field?

Ans. $16\frac{23}{24}$ acres.

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149. What is the rule for subtraction of fractions?

4. Bought a cask of wine containing 37$\frac{5}{8}$ gal.; 16$\frac{5}{6}$ gallons having leaked out, what quantity remained in the cask? Ans. 20$\frac{19}{24}$ gal.

5. A boy while fishing for pickerel, lost part of his pole; and on measuring, he found that the part saved was $\frac{5}{8}$ of the original length. What part was broken off?

6. From a chest of tea weighing 62$\frac{3}{8}$ lb. 39$\frac{4}{5}$ lb. were sold. How many pounds remain unsold?

151. MISCELLANEOUS EXAMPLES IN FRACTIONS.

1. Reduce $\frac{24}{36}$ to its lowest terms.
2. Reduce 7$\frac{2}{3}$ to an improper fraction.
3. Reduce $\frac{85}{7}$ to a mixed number.
4. Multiply $\frac{27}{63}$ by 9.
5. Multiply $\frac{87}{107}$ by 6.
6. Divide $\frac{24}{40}$ by 4.
7. Divide $\frac{65}{3}$ by 10.
8. Divide 23$\frac{1}{3}$ by 7.
9. Multiply 20 by $\frac{3}{4}$.
10. Multiply 100$\frac{2}{8}$ by $\frac{3}{8}$.
11. Multiply $\frac{9}{12}$ by $\frac{15}{16}$.
12. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{9}{18}$ to a simple fraction.
13. Divide $\frac{19}{20}$ by $\frac{6}{7}$.
14. Divide 207 by $\frac{4}{5}$.
15. Reduce $\frac{7}{8}$, $\frac{6}{12}$, $\frac{5}{24}$ to equivalent fractions having a common denominator.
16. Add $\frac{2}{7}$ and $\frac{14}{7}$.
17. Add $\frac{94}{105}$ and $\frac{1}{2}$.
18. Add 10$\frac{1}{2}$ and 6$\frac{1}{7}$.
19. Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$.
20. Subtract $\frac{2}{8}$ from $\frac{6}{8}$.
21. Subtract $\frac{1}{2}$ from $\frac{19}{20}$.

22. Subtract $\frac{47}{165}$ from 1.

23. Subtract $2\frac{1}{4}$ from $3\frac{1}{3}$.

24. Reduce $\frac{15}{25}$, $\frac{48}{108}$, $\frac{56}{96}$ to equivalent fractions with a least common denominator.

25. Reduce $\frac{75}{191}$ of $\frac{85}{206}$ to a simple fraction.

EXAMPLES IN ANALYSIS.

152. We analyze an example when we solve it according to its own conditions without being guided by any particular rule.

1. If 1 pound of tea cost $1.20, what will $\frac{4}{5}$ of a pound cost?

ANALYSIS. If 1 lb. cost \$ 2.20, $\frac{1}{5}$ of a pound will cost $\frac{1}{5}$ of \$ 1.20, or 24 cts.; and $\frac{4}{5}$ of a pound will cost 4 times 24 cts., or 96 cts., which is the answer.

NOTE. A period called the decimal point, which will be hereafter more fully explained, is placed at the right hand of dollars, and the first two places at the right of the points always express cents. In the following examples, if no cents are named with the dollars, their places may be supplied with two ciphers.

2. If 1 cord of wood cost \$ 9, what will $\frac{5}{8}$ of a cord cost? Ans. \$ $5.62\frac{1}{2}$.

3. What will $\frac{5}{7}$ of a ton of hay cost if 1 ton cost 21 dollars? Ans. \$ 15.

4. When oil is \$ 2.25 per gall. what will $\frac{2}{3}$ of a gall. cost? Ans. \$ 1.50.

5. If 1 acre of land cost \$ 140, what will $\frac{7}{11}$ of an acre cost? Ans. \$ $89.09\frac{1}{11}$.

6. If $\frac{3}{5}$ of a yd. of cloth cost \$ 2.50, what will be the cost of 1 yd.?

ANALYSIS. If $\frac{3}{5}$ of a yd. cost \$ 2.50, $\frac{1}{5}$ will cost $\frac{1}{3}$ of \$ 2.50, or \$ $.83\frac{1}{3}$, and $\frac{5}{5}$ or 1 yd. will cost 5 times \$ $.83\frac{1}{3}$, or \$ $4.16\frac{2}{3}$., which is the answer.

7. Bought $\frac{4}{5}$ of an iron foundry for \$ 6783, what was the value of the whole foundry? Ans. \$ 8478.75.

8. When we pay $62 for $\frac{7}{8}$ of an acre of land what is the cost per acre? Ans. $\$70.85\frac{5}{7}$.

9. If $\frac{7}{9}$ of a gallon of molasses cost $.83, what is the cost per gallon?

10. If 3A. 2R. 30sq. rd. is $\frac{5}{9}$ of a field, what is the entire area? Ans. 6A. 2R. 22sq. rd.

11. A grocer sold from a cask 15gal. 3qt. 1pt. of oil, which was $\frac{3}{7}$ of what the cask contained; how much remained in the cask? Ans. 21 gal. 0qt. $1\frac{1}{3}$pt.

Note. It is evident that if he had sold $\frac{3}{7}$ of what the cask at first contained, $\frac{4}{7}$ remained.

12. If 6 doz. eggs cost $1.68, what will $12\frac{3}{4}$ doz. cost?

Analysis. If 6 doz. cost $1.68, 1 doz. will cost $\frac{1}{6}$ of $1.68, or 28cts. $12\frac{3}{4}$ doz. $=\frac{51}{4}$ doz. If 1 doz. cost 28cts. $\frac{1}{4}$ doz. will cost $\frac{1}{4}$ of 28cts. or 7cts., and $\frac{51}{4}$ will cost 51 times 7cts., or $3.57. Ans.

13. If 5 rods of wall can be built for $8.65 what will it cost to build $17\frac{3}{8}$ rods? Ans. $\$30.05\frac{7}{8}$.

14. If 16 bushels of corn cost $22.72, what will $47\frac{2}{3}$ bushels cost? Ans. $\$67.68\frac{2}{3}$.

15. If a family consume 4 barrels of flour in $7\frac{1}{2}$ months, how long would $9\frac{1}{3}$ barrels last them? Ans. $17\frac{1}{2}$ months.

16. If $\frac{7}{8}$ gal. wine cost $4.75, what will $6\frac{4}{5}$ gal. cost?

Analysis. If $\frac{7}{8}$ gallons of wine cost $4.75, $\frac{1}{8}$ will cost $\frac{1}{7}$ of $4.75 or $\$.67\frac{6}{7}$, and $\frac{8}{8}$ or 1 gallon will cost 8 times $67\frac{6}{7}$cts. or $\$5.42\frac{6}{7}$. $6\frac{4}{5}$ gallons $=\frac{34}{5}$ gallons. Now if 1 gallon cost $\$5.42\frac{6}{7}$, $\frac{1}{5}$ gallon will cost $\frac{1}{5}$ of $\$5.42\frac{6}{7}$, or $\$1.08\frac{4}{7}$, and $\frac{34}{5}$ will cost 34 times $\$1.08\frac{4}{7}$ or $\$37.33\frac{5}{7}$. Ans.

17. When $\frac{6}{7}$ of a dollar will purchase 3 qts. of cherries, how many can you purchase for $2\frac{4}{5}$ dollars? Ans. $9\frac{4}{5}$ qt.

18. If you can buy $4\frac{2}{3}$ tons of hay for \$ 70, what will $12\frac{5}{9}$ tons cost? Ans. \$ $188\frac{1}{3}$.

19. When $17\frac{1}{2}$ cents are paid for $1\frac{3}{4}$ lbs. of nuts, how many pounds will $48\frac{3}{4}$ cents buy? Ans. $4\frac{7}{8}$ lbs.

20. If $6\frac{2}{3}$ bushels of wheat cost \$15, how many bushels can you buy for $68\frac{5}{6}$ dollars?

21. Sold $5\frac{3}{8}$ cords of wood to one man, and $12\frac{4}{7}$ cords to another; how much did I sell to both?

22. How many hours will it take a man to travel $65\frac{2}{5}$ miles, if he travel $3\frac{3}{8}$ miles in an hour? Ans. $19\frac{17}{45}$ h.

23. Bought a horse for \$$176\frac{1}{4}$, and a wagon for \$ $67\frac{3}{8}$; how much more did the horse cost than the wagon?

Ans. \$ $108\frac{7}{8}$.

24. Paid \$ 16 for some cloth, at the rate of \$ $\frac{4}{5}$ per yd.; how many yards were there? Ans. 20 yards.

25. If 4 doz. oranges cost $\frac{3}{5}$ of \$ 4, what will 7 oranges cost? Ans. \$ $\frac{7}{20}$.

26. A man who owned $\frac{7}{9}$ of a farm sold $\frac{3}{5}$ of his share; what part of the farm did he sell and what part did he still own? Ans. sold $\frac{7}{15}$, and had left $\frac{14}{45}$.

27. If a man has $28\frac{4}{7}$ gal. of wine, and sells $\frac{3}{4}$ of it, how many gallons has he left? Ans. $7\frac{1}{7}$.

28. A man has 7 pieces of broadcloth, each piece containing $26\frac{4}{5}$ yd. This he makes up into overcoats requiring $4\frac{2}{3}$ yards each; how many garments can he make and how much cloth will he have left? Ans. 40 garments, and $\frac{14}{15}$yd. left.

29. A field containing $157\frac{1}{2}$sq. rd. is $9\frac{3}{8}$rd. wide; how long is it? Ans. $16\frac{4}{5}$ rd.

30. If $\frac{5}{6}$ of a ton of coal can be bought for \$ 7, what part of a ton can be bought for one dollar? Ans. $\frac{5}{42}$.

31. If $\frac{7}{9}$ of a ton of hay is worth \$ $12\frac{1}{4}$, what is the value of a ton? Ans. \$ $15\frac{3}{4}$.

32. One man earns \$ $1\frac{5}{8}$ per day, and another earns \$ $2\frac{1}{5}$; how much will they both earn in 5 days?

Ans. \$ $19\frac{1}{8}$.

33. A merchant buys flour at \$ $9\frac{5}{8}$ per barrel and sells it for \$ $12\frac{4}{5}$; how much will he make on 5 barrels?

Ans. \$ $15\frac{7}{8}$.

34. A tailor made 3 suits of clothes, each containing $3\frac{4}{7}$ yards of cloth, from a piece 35 yards long; how much was left? Ans. $24\frac{2}{7}$ yd.

35 What will $12\frac{3}{4}$ cords of wood cost at \$ $8\frac{5}{8}$ per cord?

Ans. \$ $109\frac{31}{32}$.

36. A farm containing 247 acres was sold for \$ $21118\frac{1}{2}$; what was the price per acre? Ans. \$ $85\frac{1}{2}$.

37. Four partners purchase goods to the amount of \$ $1264\frac{5}{8}$, and sell the same for \$ $1586\frac{3}{5}$. The profits being divided equally, what was each one's share?

Ans. \$ $80\frac{79}{160}$.

38. A tailor paid \$ $12\frac{4}{5}$ for cloth, and \$ $5\frac{3}{4}$ for making up the same into a coat, vest, and pants, and sold the same for \$ $26\frac{1}{4}$; what were his profits? Ans. \$ $7\frac{7}{10}$.

39. A merchant sold to a customer $5\frac{3}{8}$yd. broadcloth, $6\frac{1}{5}$yd. doeskin, and $4\frac{3}{4}$yd. cassimere; how many yards in all were there? Ans. $16\frac{13}{40}$.

40. When flour is \$ 15 per bbl. how many barrels can be bought for $46\frac{1}{2}$ bushels of wheat at \$ $3\frac{1}{8}$ per bushel?

Ans. $9\frac{11}{16}$ bbl.

41. A person owning $\frac{1}{2}$ of a ship, sold $\frac{3}{16}$ of his share for \$ 6000, which was \$ 950 more than it cost him; what did he pay for his entire share?

42. A gentleman has \$ 9750 invested in United States Bonds, which is $\frac{2}{5}$ of his fortune; how much is he worth?

DECIMAL FRACTIONS.

153. A DECIMAL FRACTION is a fraction whose denominator is 10, 100, 1000, or 1 with one or more ciphers annexed.

154. The denominator of a *Common Fraction* may be *any number whatever.* Every principle and every operation in Common Fractions is equally applicable to Decimals.

155. The denominator of a decimal fraction is not usually expressed, since it can be easily determined, it being 1 *with as many ciphers annexed as there are figures in the given decimal.*

156. A decimal fraction is distinguished from a whole number by a *period,* called the *decimal point* or *separatrix,* placed before the decimal; the first figure at the right of the point is *tenths;* the second, *hundredths*; the third, *thousandths;* etc.; thus, $.6 = \frac{6}{10}$, $.06 = \frac{6}{100}$, $.006 = \frac{6}{1000}$, etc., the figures in the decimal decreasing in value from left to right, as in whole numbers.

157. Since whole numbers and decimal fractions both decrease by the same law from left to right, they may be expressed together in the same example, and numerated as in the following

NUMERATION TABLE.

Thousands,	Hundreds,	Tens,	Units,	*Decimal Point,*	Tenths,	Hundredths,	Thousandths,	Ten-Thousandths,	Hundred-Thousandths,	Millionths.
3	4	7	1	.	6	5	9	8	7	2

158. A whole number and decimal fraction written together, as in the above table, form a *mixed number*. The integral part is numerated from the decimal point towards the *left*, and the fraction from the same point towards the *right*, each figure, both in the whole number and decimal, ***taking its name and value from its distance from the decimal point.*** Hence,

159. Moving the decimal point one place towards the *right*, ***multiplies*** the number by 10; moving the point two places multiplies the number by 100, etc. Also moving the point one place to the *left*, ***divides*** the number by 10; moving the point two places divides by 100, etc.

160. A decimal is read like a whole number, giving in addition, the name of the denomination of the right-hand figure to the entire number. Thus, .46 is read forty-six hundredths; .073 is read seventy-three thousandths; .0068 is read sixty-eight ten thousandths; 42.045 is read forty-two, and forty-five thousandths, etc.

161. Since multiplying both terms of a fraction by the same number does not alter its value (Art. 139, 5th), ***annexing*** one or more ciphers to a decimal does not affect its value; thus, $\frac{2}{10} = \frac{20}{100} = \frac{200}{1000}$, etc.; i. e. .2 = .20 = .200, etc.

162. *Prefixing* a cipher to a decimal, i. e. inserting a cipher between the separatrix and a decimal figure, ***diminishes***

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153. What is a Decimal Fraction? **154.** A Common Fraction, what is its denominator? Are the principles of common fractions applicable to decimals? **155.** Is the denominator of a decimal usually expressed? **156.** How is a decimal fraction distinguished from a whole number? What is the first figure at the right of the point? Second? Third? **157.** Read the Numeration Table. **158.** What is a mixed number? Which way is the integral part numerated? Which way the decimal? What determines the name and value of a figure? **159.** How does moving the decimal point to the *right* affect the value of a number? How moving it to the *left*? **160.** How is a decimal read? Illustrate. **161.** How does annexing one or more ciphers to a decimal affect it? Why? **162.** Prefixing one or more ciphers to a decimal how affects it? Why?

the value of that figure to $\frac{1}{10}$ its previous value; for it removes the figure one place further from the decimal point (Art. 159); thus, .3 = $\frac{3}{10}$ but .03 = only $\frac{3}{100}$, which is but $\frac{1}{10}$ of $\frac{3}{10}$.

NOTATION AND NUMERATION OF DECIMAL FRACTIONS.

163. Let the pupil write in figures the following numbers:

1. Fifty-six hundredths.
2. Eighty-seven thousandths.
3. Two hundred sixteen ten-thousandths.

NOTE. To avoid ambiguity in expressing a whole number and a decimal, we should use "*and*" *only once*, and that, between the whole number and decimal, as two hundred three, and six thousandths, (203.006); *not*, two hundred *and* three *and* six thousandths. By observing the above rule, much trouble will be obviated, and numerous mistakes avoided.

4. Twenty-eight, and one thousand nine, ten-thousandths.
5. One hundred sixty-eight, and thirteen millionths.
6. Eight hundred forty, and forty-two hundred thousandths.
7. Seven, and seven hundredths.
8. Four hundred eighty-seven, and three hundred forty-four ten-thousandths.
9. Sixteen thousand, four hundred thirty nine, and ninety-two thousandths.
10. Fifty-one million two thousand eighty-five, and seventeen hundredths.
11. Four thousand eight hundred twenty-eight, and nine hundred fifty-six hundred-thousandths.
12. Eighty-seven thousand three hundred forty-nine, ten-millionths.

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163. How are operations in decimal fractions performed? What of the the decimal point? Proof?

Note 2. Let the teacher give many examples, similar to the above, continuing the exercise till the pupil can write decimals with ease and accuracy.

Numerate and read the following:

1. .7
2. .03
3. 40.6
4. 601.75
5. 77.7
6. 9005.847
7. .0001
8. 1000.1
9. 45678.951
10. .37558
11. 4587.9506
12. .000001
13. 17.75851
14. 7.805
15. 84591.57
16. 42.2222
17. 10000.001
18. 45671.3501
19. 1000.001
20. 1846561.07

As all the operations in decimal fractions are performed precisely as the same operations in whole numbers, no explanations are necessary, except to determine the true place of the decimal point in the several results. The methods of proof, also, are the same as in whole numbers.

Case 1.

164. To add decimal fractions:

Rule. *Place tenths under tenths, hundredths under hundredths, etc.; then add as in whole numbers, and place the point in the sum directly under the points in the numbers added.*

	Ex. 1.		2.
	4.37		469.037
	65.082		6093.008406
	9.09		506.90005
	463.0804		7900.056209
Sum,	541.6224	Sum,	14969.001665
Proof,	541.6224	Proof,	14969.001665

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164. Rule for addition of decimals?

3. Add 469.05309, 27.039, 8056.00963.
Ans. 8552.10172.

4. Add 904.0602, 6095.8095, 600.06, and 29076.004069.
Ans. 36675.933769.

5. Add 2307.085063, 65.0047, 380.30027, and 70580.060309.
Ans. 73332.450342,

6. Add 4.063, 85.605, 74608.37, 63.704.

7. Add two hundred forty-three, and sixty-five thousandths; seventy-one, and eighty-four ten-thousandths; two thousand, and two thousandths; six hundred, and six hundredths.
Ans. 2914.1354.

8. Add six hundred fifty-eight, and seven hundred two ten-thousandths; ninety-seven, and ninety-seven hundredths; two thousand sixty-five, and eight thousand three hundred six hundred-thousandths.

9. Add sixty-eight millionths; two hundred forty-seven ten-thousandths; nine hundred seventy-two hundred-thousandths.
Ans. .034488.

10. Add three thousand nine hundred sixty-two, and seven hundred thousandths; five hundred seventy-three, and ninety-three ten-thousandths; eight thousand forty-four, and seven hundredths.

CASE 2.

165. To subtract a less decimal from a greater:

RULE. *Place the less number under the greater, tenths under tenths, etc.; then subtract as in whole numbers, and place the point in the remainder, directly under the points in the minuend and subtrahend.*

---

165. Rule for subtraction of decimals? When the number of decimal places in the subtrahend exceeds the number of decimal places in the minuend what is done?

	Ex. 1.	2.	3.
	5.923	53.0876	4963.0032
	2.867	47.1984	874.085769
Rem.	3.056	5.8892	4088.917431
Proof,	5.923	53.0876	

Note. Whenever there are more decimal figures in the subtrahend than in the minuend, as in Ex. 3, we may supply the deficiency by annexing ciphers, or *supposing* them annexed, to the minuend.

4. From 68.0473 take 39.0027. Ans. 29.0446.

5. From 234.0023 take 97.013005.

6. From 608.01004 take 290.020635.
Ans. 317.989405.

7. From 5901.632 take 807.000956.
Ans. 5094.631044.

8. From 20.006 take 7.020407.

9. From one hundred eighty-three, and twenty-four thousandths, take seventy-six, and three thousand seven hundred ninety-eight ten-millionths. Ans. 107.0236202.

10. From five thousand six hundred nine, and one hundred thirty-two hundred-thousandths, take nine hundred eighty-five, and four hundred ninety-six ten-thousandths.

Case 3.

166. To multiply one decimal by another:

Rule. *Multiply as in whole numbers, and point off as many figures for decimals in the product as there are decimal places in both factors counted together.*

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166. Rule for multiplication of decimals? Reason of the rule for pointing the product? Suppose there are not figures enough in the product for observing the direction for pointing off?

Ex. 1. Multiply .37 by .28.

	OPERATION.	PROOF.
Multiplicand,	.3 7	
Multiplier,	.2 8	
	2 9 6	$\frac{37}{100} \times \frac{28}{100} = \frac{1036}{10000}$.
	7 4	
Product,	.1 0 3 6	

Note. 1. The reason of the rule for pointing the product will be obvious if we change the decimals to the form of common fractions and then perform the multiplication; hence, the proof above.

Thus we have, $.24 \times .16 = \frac{24}{100} \times \frac{16}{100} = \frac{384}{10000} = .0384$.

And again, $.0069 \times .09 = \frac{69}{10000} \times \frac{9}{100} = \frac{621}{1000000} = .000621$.

2. Multiply 3.0628 by 7.4.

OPERATION.	PROOF.
3.0 6 2 8	$3.0628 = 3\frac{628}{10000} = \frac{30628}{10000}$
7.4	$7.4 = 7\frac{4}{10} = \frac{74}{10}$
1 2 2 5 1 2	$\frac{30628}{10000} \times \frac{74}{10} = \frac{2266472}{100000}$
2 1 4 3 9 6	
2 2.6 6 4 7 2	$\frac{2266742}{100000} = 22.\frac{66472}{100000} = 22.66472$. Ans.

3. Multiply .0638 by .83. Ans. .052954.

Note 2. If the number of figures in the product is less than the number of decimal places in the two factors, the deficiency must be supplied by *prefixing ciphers to the product*, as in Ex. 3.

4. Multiply 56.029 by .08506 Ans. 4.76582674.
5. Multiply 289.406 by 56.09. Ans. 16232.78254.
6. Multiply 368.09203 by 8.43.
7. Multiply 14.063 by 10. Ans. 140.63.

Note. 3. By referring to Art. 159, it will be seen that removing the decimal point one place to the right multiplies the number by 10, etc.

8. Multiply .0863 by 300.

9. Multiply 28.07 by .08 Ans. 2.2456.

10. Multiply 4.7306 by 2.09.

11. Multiply 97.084 by .063. Ans. 6.116292.

12. Multiply .75 by .0024. Ans. .0018.

13. Multiply 803.006 by .0001.

14. Multiply .06053 by .0057. Ans. .000345021.

15. Multiply 119.79325 by .006. Ans. .7187595.

16. Multiply 68.003 by 8.04.

17. Multiply 8.59 by 240. Ans. 2061.6.

18. Multiply .06 by 0003.

19. Multiply .863 by 1000. Ans. 863.

20. Multiply 3800 by .046. Ans. 174.8.

21. Multiply 6000 by .006.

22. Multiply .07 by .07, also .5 by .5.

23. If the multiplicand is 642. 08069, and the multiplier is 46.008, what is the product?

24. If a man can earn $ 64.925 in 1 month, how much can he earn in 8.4 months? Ans. $ 545.37.

25. If 1 barrel of potatoes weighs 124.8 lb., how much will 28.5 barrels weigh? Ans. 3556.8 lb.

26. If a trader gains $.0625 on one pound of tea, what will he gain on 3000 pounds?

27. Should the same trader lose $.875 on 1 bbl. flour, what would be his loss on 500 barrels? Ans. $ 437.50.

28. If it require 6.75 yards of cloth to make a uniform for a soldier, how many yards will it take to furnish 4 regiments, of 1000 men each?

29. If a horse will travel 46.875 miles in one day, how far will he travel in four weeks, resting on each Sabbath? Ans. 1125 miles.

30. When $16.5 are paid for 1 ton of hay, what will 6.75 tons cost? Ans. $ 111.375.

CASE 4.

167. To divide one decimal by another:

RULE. *Divide as in whole numbers, and point off as many figures for decimals in the quotient as the number of decimal places in the dividend exceeds those in the divisor.*

Ex. 1. Divide 5.12 by .8.

OPERATION.

$$\begin{array}{r} .8\,)\,5.1\,2\,(\,6.4 \\ \underline{4\,8} \\ 3\,2 \\ 3\,2 \end{array}$$

PROOF.

The mixed number $5.12 = \frac{512}{100}$, and, $.8 = \frac{8}{10}$.

$$\frac{512}{100} \div \frac{8}{10} = \frac{\overset{64}{\cancel{512}}}{\underset{10}{\cancel{100}}} \times \frac{\cancel{10}}{\cancel{8}} = \frac{64}{10} = 6.4.$$

NOTE 1. The rule for determining the place of the point in the quotient may be explained by changing the decimals to the form of common fractions and performing the division.

2. Divide .000048 by .03. Ans. .0016.

NOTE 2. If the number of figures in the quotient is less than the excess of decimal places in the dividend over those of the divisor, supply the deficiency by *prefixing ciphers to the quotient.*

3. Divide 420.075 by 25. Ans. 16.803.
4. Divide .19872 by .276. Ans. .72.
5. Divide 34.944 by 96.
6. Divide 36.75 by .25.
7. Divide .04212 by 4.68. Ans. .009.
8. Divide 1167.25 by 287.5. Ans. 4.06.
9. Divide 44.8514 by 7.03. Ans. 6.38.

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167. Rule for division of decimals? How may the method of determining the place of the decimal point in the quotient be explained? If the number of figures in the quotient is less than the excess of the decimals in the dividend over those in the divisor, what is to be done? How may the decimal places of dividend and divisor be made equal? What is then the quotient? How is a remainder sometimes indicated?

10. Divide .06 by .0002. Ans. 300.

11. Divide .5 by .125. Ans. 4.

NOTE 1. If there are more decimal places in the divisor than in the dividend, the number may be made equal by annexing one or more ciphers to the dividend. The quotient will then be a whole number.

12. Divide 43.6 by 7.2. Ans. 6.+.

NOTE 2. When a remainder occurs in the division, we sometimes write the sign + after the quotient, to show that the decimal is not complete, or we may annex the remainder in the form of a common fraction.

13. Divide .875 by .42. Ans. 2.0$\frac{5}{6}$.

14. Divide 46.71 by 2.3. Ans. 20.3 +.

15. Divide 58.996 by 4.5. Ans. 13.11+.

16. Divide 875 by .875.

17. Divide 4. by .16.

18. Divide .16 by 4.

19. 28.75 ÷ 2.5. Ans. 11.5.

20. 436.8 ÷ .74. Ans. 590 +.

21. .08625 ÷ .005. Ans. 17.25.

22. If 400 barrels of flour cost $ 6700, what will one barrel cost? Ans. $ 16.75.

23. A grocer sold a quantity of tea; he gained $.125 on a pound, and his whole gain was $ 3.75, how many pounds did he sell? Ans. 30.

24. If a chest of tea holds 63.75 lb., how many chests will it require to hold 1912.5 lb.? Ans. 30.

25. If a steamer runs 617.5 miles in 32.5 hours, how far does she go in 1 hour? Ans. 19 miles.

26. If a family consume 6.5 barrels of flour in one year, how long would 317.85 barrels last the same family?

27. A man paid $ 262.30 for 61 sheep, how much did he apiece? Ans. $ 4.30.

CASE 5.

168. To reduce a common fraction to a decimal:

Ex. 1. Reduce $\frac{5}{8}$ to a decimal fraction.

If we multiply a fraction by any number and then divide by the multiplier, the quotient will be the multiplicand. Accordingly, in the above example, we multiply $\frac{5}{8}$ by 1000 = $\frac{5000}{8}$ = 625; and 625 ÷ by 1000 = $\frac{625}{1000}$ = .625; and hence we have the following

RULE. *Annex one or more ciphers to the numerator and divide the result by the denominator, continuing the operation until there is no remainder, or as far as is desirable. Point off as many decimal places in the quotient as there are ciphers annexed to the numerator.*

2. Reduce $\frac{3}{4}$ to a decimal.
$\frac{3}{4} \times 100 = \frac{300}{4} = 75$; and $75 \div 100 = .75$ Ans.
3. Reduce $\frac{9}{16}$ to a decimal. Ans. .5625.
4. Reduce $\frac{75}{64}$ to a decimal. Ans. 1.171875.
5. Reduce $\frac{85}{32}$ to a decimal.
6. Reduce $\frac{5}{12}$ to a decimal. Ans. .4166 +.
7. Reduce $\frac{1}{2}$, $\frac{3}{5}$, $\frac{2}{3}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{11}{12}$ to decimals.

CASE 6.

169. To reduce a decimal to a common fraction:

Ex. 1. Reduce .75 to a common fraction. .75 = $\frac{75}{100}$, and this, reduced to its lowest terms = $\frac{3}{4}$ Ans. Hence,

RULE. *Write the denominator to the decimal, omitting the decimal point, and then reduce the common fraction to its lowest terms.*

2. Change .625 to a common fraction. Ans. $\frac{5}{8}$.

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168. Rule for reducing a common fraction to a decimal? **169.** Rule for reducing a decimal to a common fraction?

3. Change .375 to a common fraction and to its lowest terms. Ans. $\frac{3}{8}$.

4. Reduce .0625 to a common fraction. Ans. $\frac{1}{16}$.

5. What common fraction is equivalent to .4375? Ans. $\frac{7}{16}$.

6. Reduce .68 to a common fraction.

7. Change .875 to a common fraction.

8. Change .0075 to a common fraction.

Miscellaneous Examples in Decimals.

1. What is the sum of one-tenth, one hundredth, and forty-seven thousandths? Ans. .157.

2. What is the difference between seven hundredths and eight thousandths? Ans. .062.

3. .065 — .0098 = what?

4. Multiply eighty-four hundredths by forty-seven ten-thousandths. Ans. .003948.

5. Divide two by four-tenths.

6. Divide eighteen thousandths by six millionths. Ans. 3000.

7. From seven-tenths take four millionths. Ans. .699996.

8. Paid $ 480 for a piece of land at $ 62.50 per acre; how many acres were there? Ans. 7.68.

9. What cost 8 acres of land at $ 68.75 per acre?

10. What cost 6.75 lb. of coffee at $ .24 per lb.?

11. How many casks each holding 37.5 gallons can be filled with 1687.5 gallons of wine?

12. Bought 6.5 tons hay at $ 16.875 per ton; what was the entire cost? Ans. $ 109.6875.

13. What common fraction is equal to the sum of .625 and .0625? Ans. $\frac{11}{16}$.

14. When $ 18.5625 is paid for 148.5 yds. of cloth, what is the cost per yard?

15. What will 17 pairs of boots cost at $ 10 875 per pair?

16. Change .68 to a common fraction. Ans. $\frac{68}{100} = \frac{17}{25}$.

17. Change 5.25 to a common fraction.

Ans. $\frac{525}{100} = \frac{21}{4} = 5\frac{1}{4}$.

18. How many pairs of shoes at $ 1.25 can be purchased for $ 45?

19. How many pounds of sugar at $ .18 per pound can you buy for $ 6.12? Ans. 34.

20. What will 12 bales of cotton cost, each bale weighing 5.25 cwt. at $ 46.50 per cwt.?

21. If .625 of a ton of coal cost $ 5.75, what will one ton cost? Ans. $ 9.20.

22. What cost .875 of a ton of coal at $ 12 per ton?

Ans. $ 10.50.

23. What will 8.75 cords of wood cost at $ 10 per cord?

24. If a boat will sail 7.5 miles in 1 hour, how far will she sail in 9.75 hours?

25. Divide one hundred by one hundredth.

26. Multiply one thousandth by one thousandth.

27. If 9564.75 rods of wall can be built in 87.75 days, how many rods can be built in one day? Ans. 109.

28. I have a room 15.50 feet wide, 16.75 feet long; how many square feet does the floor contain? Ans. 259.625.

29. How many square yards of carpeting would it take to carpet the above room?

30. How much would the above carpet cost at $ 1.75 per yard?

31. A load of hay weighs 1675.25 lb. how much will it cost at $ 2.50 per cwt.? Ans. $ 41.88.

32. A ship carries 725 bales of cotton, each bale weighing 400 pounds; how much will the freight amount to at $.0125 per pound?

UNITED STATES MONEY.

170. United States Money, sometimes called *Federal Money*, is the currency of the United States.

TABLE.

10 Mills (m.)	make	1 Cent,	Marked	c.
10 Cents	"	1 Dime,	"	d.
10 Dimes	"	1 Dollar,	"	$
10 Dollars	"	1 Eagle,	"	e.

						Cents.		Mills.
				Dimes.		1	=	10
		Dollars.		1	=	10	=	100
Eagle.		1	=	10	=	100	=	1000
1	=	10	=	100	=	1000	=	10000

Note. The terms eagle and dime are seldom or never used in computation; eagles and dollars being read collectively and called dollars, and dimes and cents being called cents; thus, 3 eagles and 5 dollars are called $35, and 4 dimes and 3 cents are called 43 cents. When *mills* are written with dollars and cents they are set in the *third place* at the right of the period; thus, thirty-five dollars, forty-three cents, and seven mills, expressed in figures, is $35.437.

171. A *coin* is a piece of gold, silver, or other metal, stamped by authority of the General Government, to be used as money.

172. The coins authorized by our Government, and stamped at the U. S. Mint, are the following:

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170. What is United States Money? Repeat the Table. Are the terms eagle and dime much used? How are eagles and dollars read? Dimes and [illegible]s? What place do mills occupy? Illustrate. **171.** What is a coin?

GOLD.		SILVER.	
Double Eagle,	$20.00	Trade Dollar,	$1.00
Eagle,	10.00	Half Dollar,	.50
Half Eagle,	5.00	Quarter Dollar,	.25
Quarter Eagle,	2.50	Dime,	.10
Three Dollar Piece,	3.00		
One Dollar,	1.00		

Also of copper and nickel, we have the One Cent, Three Cent, and Five Cent Pieces, called minor coins.

NOTE 1. The *mill* is not coined.

NOTE 2. The silver half-dime, three-cent piece, old silver dollar and bronze two-cent piece are no longer coined. The trade dollar is especially designed for the China and Japan trade.

NOTE 3. The greater part of the currency in general use in this country consists of Treasury Notes and Fractional Currency, issued by Government, and Notes issued by the National Banks.

Operations in United States Money are performed precisely like those in Decimal Fractions, the dollar being considered the UNIT. Therefore no special rules are needed.

PRACTICAL EXAMPLES.

1. Paid $ 12.50 for a barrel of flour, $ 2.375 for a box of sugar, $ 17.875 for a tub of butter, and $ 5.25 for a cheese; what did I pay for all? Ans. $ 38.

OPERATION.

$$\begin{array}{r} \$12.50 \\ 2.375 \\ 17.875 \\ 5.25 \\ \hline \$38.000 \end{array}$$ Ans.

Having set dollars under dollars, cents under cents, etc., *add* as in Art. 164, and set the sum below, remembering that the *point*, or period, should be placed directly under the points in the numbers added.

---

172. What gold coins are authorized by our Government? What silver coins? What other coins? What is said of the mill? What of the trade dollar? What of paper money? How are operations in U. S. Money performed?

2. Bought a coat for $ 21.75, a vest for $ 5.35, a pair of pantaloons for $ 8.40, a hat for $ 5.25, a pair of boots for $ 7.50, and various other articles for $12.75; what must I pay for all?

3. A farmer paid $ 125.50 for a pair of oxen, $ 52 for a cow, $ 350.75 for a horse, and $ 45.25 for a harness; how much did all cost? Ans. $ 573.50.

4. A merchant in returning from the city found he had expended $ 1050.375 for dry goods, $ 850.75 for groceries and $ 250.875 for hardware; what was the amount of his purchases?

5. A broker has $ 19505.00 in one bank, $ 4550.50 in another bank, and $ 6750.37 in another; how much money has he in the three banks? Ans. $ 30805.87.

6. The property of a gentleman is divided as follows; he has $ 5750 in bank stock, $ 3100.50 in notes at interest, a manufactory worth $ 19587, two farms, one worth $ 5780 and the other twice as much, and $ 6850.75 due him on accounts; how much is he worth?

7. A man who owed $ 87.37, paid $ 16.52; how much did he still owe?

OPERATION.

$$\begin{array}{r} 87.37 \\ 16.52 \\ \hline \$\,70.85 \end{array}$$ Ans.

Having set the less number under the greater, dollars under dollars, etc., *subtract* as in Art. 165, remembering to place the point in the remainder under the points in the minuend and subtrahend.

8. Paid $ 175.625 for a pair of oxen, and $ 132.375 for a horse; how much more did I pay for the oxen than for the horse? Ans. $ 43.25.

9. A gentleman purchased a city residence for $ 19570, which was $ 8957.75 more than his country place cost him; what did his country place cost him?

10. A drover paid out for cattle $ 5767.50; he received he same lot, besides expenses of taking them to market,); how much were his profits? Ans. $ 762.50.

11. A merchant went to the city to buy goods, with $ 3575.50 in cash; he bought to the amount of $ 5050; how much did he buy on credit? Ans. $ 1474.50.

173. To find the cost of any number of things when the price of one thing is given.

12. Bought 6 cows at $ 35.375 each; what did I pay for the lot?

OPERATION.
$ 35.375
6
$ 212.250, Ans.

Six cows will evidently cost 6 times as much as one cow. All similar examples are solved in like manner. Hence, the following

RULE. *Multiply the price of one by the number.*

13. What is the cost of 7 barrels of flour at $ 8.50 per barrel?

14. Bought 16 yards of silk, at $ 1.75 per yard; what was the cost of the piece? Ans. $ 28.00.

15. Bought 33 sheep, at $ 8.25 per head; what was the cost of the flock? Ans. $ 272 25.

16. What are 85 pounds of butter worth, at 37 cents per pound? Ans. $ 31.45.

17. What are 625 cords of wood worth at $ 8.75 per cord? Ans. $ 5468.75

18. What is a cargo of coal of 2070 tons worth at $ 11.25 per ton? Ans. $ 23287.50.

19. What are 1625 bushels of wheat worth at $ 3.75 per bushel? What would be the freight on the above at $.625 per bushel?

20. Supposing the above wheat to make 425 barrels of flour, how much would it be worth at $ 17.50 per barrel? Ans. $ 7437.50.

---

173. How can you find the cost of any number of things when you know the price of one?

174. To find the price of an article when the cost of a given number of articles is known.

21. Paid $1129.50 for 9 horses; what was the average price per horse?

OPERATION.

9) 1129.50

$125.50, Ans.

One horse is worth *one ninth* as much as 9 horses. To obtain one-ninth of any number we divide the number by 9. Hence, the

RULE. *Divide the cost by the number.*

22. Paid $19.61 for 53 pounds of butter; what was the price per pound? Ans. 37 cents.

23. If I pay $315.75 for 25 barrels of flour, what is the price per barrel? Ans. $12.63.

24. If a mechanic earns $65.24 in 28 days, what is his daily wages?

25. Paid $74.75 for 13 weeks' board; what was the price per week? Ans. $5.75.

26. Seventy-seven boys paid $2233 for 1 year's tuition; what did each boy pay? Ans. $29.

27. Bought a farm containing 87 acres for $4763.25; what was the price per acre?

28. If 8 barrels of flour cost $75, what is the price per barrel? Ans. $9.375.

OPERATION.

8) $75.000

$9.375, Ans.

When the division is incomplete and there are no cents and mills in the dividend, ciphers may be annexed to the dividend and the division continued.

---

174. How can you find the price of one article when you know the cost of 'ven number of articles? Explain Ex. 28.

PROOF.
$9.375
8
$75.000

Reversing the above process, as in the *proof*, will cause the cents and mills to disappear and bring back the original dividend.

29. If 24 men earn $63 in a day, what will 1 man earn in the same time? Ans. $2.625.

30. Paid $6300 for 36 horses; what was the price of each?

31. If 5 barrels of flour are worth $47.267, what is 1 barrel worth? Ans. $9 453+.

32. Paid $34.88 for 9 yards of cloth; what was the price per yard? Ans. $3.875+.

33. If a cargo of wood is worth $19275, and the number of cords is 2850, how much is the price per cord?
Ans. $6.76+.

34. If $1000 will buy 850 bushels of corn, what is the price per bushel? Ans. $1.17+.

35. If a merchant's bill for flour was $18500 in one month, and he purchased 1500 barrels, what did it cost him, on an average, per barrel?

175. To find the quantity when the cost of the quantity and the price of one are given.

36. At $8 a ton, how many tons of coal can I buy for $240?

OPERATION.
$8)$240
30, Ans.

As many times as $8 is contained in $24 so many tons I can buy. Hence, the

RULE. *Divide the cost by the price of one.*

---

175. How do you find the quantity when the total cost and the price of one are given?

37. At 16 cents a pound, how many pounds of sugar can I buy for \$ 19.96 ? Ans. $124\frac{3}{4}$.

38. How many yards of cloth, at \$ 2.56 per yard, can I buy for \$ 642.56 ? Ans. 251.

39. How many sheep, at \$ 7.75 a head, can be bought for \$ 193.75 ? Ans. 25.

40. A farmer paid \$ 3562.50 for land, at \$ 37.375 per acre; how many acres did he buy?

41. A merchant paid \$ 4498.83 for a lot of broadcloth; the average price per yard was \$ 3.33; how many yards were there? Ans. 1351.

42. How many books at \$1.75 each, can be bought for \$ 2625 ?

43. An agent has \$ 925 with which to purchase flour; at \$ 12.25 per barrel how many whole barrels can he buy? How much money will he then have left?

176. To find the cost or value of any number of articles when the price of one is an exact or aliquot part of a dollar.

TABLE OF ALIQUOT OR EXACT PARTS OF A DOLLAR.

50 cents $= \frac{1}{2}$ of a dollar,
$33\frac{1}{3}$ cents $= \frac{1}{3}$ of a dollar,
25 cents $= \frac{1}{4}$ of a dollar,
20 cents $= \frac{1}{5}$ of a dollar,
$16\frac{2}{3}$ cents $= \frac{1}{6}$ of a dollar,
$12\frac{1}{2}$ cents $= \frac{1}{8}$ of a dollar.

44. What cost 45 yards of calico, at $33\frac{1}{3}$ cents per yard?

$33\frac{1}{3}$ cents is $\frac{1}{3}$ of a dollar; hence, 45 yards will cost \$ $45 \div 3 =$ \$ 15, Ans.

45. What cost 84 pounds of butter, at 50 cents a pound?

46. What cost 48 pounds of honey, at 25 cents a pound?

47. What cost 32 bushels of corn, at $87\frac{1}{2}$ cents per bushel?

---

176. What is an aliquot part? Repeat the table of aliquot or exact parts of a dollar.

OPERATION.

$ 32 = cost of 32 bush., at $ 1.
16 = cost of 32 bush., at 50 c., or ½ of $ 1.
8 = cost of 32 bush., at 25 c., or ½ of 50c.
4 = cost of 32 bush., at 12½ c., or ½ of 25c.
Ans. $ 28 = cost of 32 bush., at 87½ c.

That is, the cost at $ 1 is evidently as many dollars as there are bushels; the cost at 50c., is half as much as at $ 1; the cost at 25c., half as much as at 50c.; and the cost at 12½c., half as much as at 25c. Then the cost at 50c., at 25c., and at 12½c., added, gives the cost at 87½c.

48. What is the value of 736 yards of gingham, at 37½ cents a yard? Ans. $ 276.

49. What shall I pay for 1832 bushels of oats, at 62½ cents per bushel?

This process is usually called *Practice*, for which we have the following

RULE. *Take such aliquot parts of the number of articles as the price is an aliquot part of* **$1.**

50. What cost 24 barrels of apples, at $ 3.75 per barrel?

OPERATION.

$ 24 = cost at $ 1.
3
$ 72 = cost at $ 3.
12 = cost at .50 or ½ of $ 1.
6 = cost at .25 or ½ of 50c.
Ans. $ 90 = cost at $ 3.75.

51. What are 348 barrels of flour worth, at $ 9.87½ per barrel? Ans. $ 3436.50.

52. What are 165 thousand of brick worth, at $ 11.75 per thousand?

---

176. How do you find the cost of any number of articles when the price is an aliquot part of a dollar? What is this process called?

53. What are 84 cases of merchandise worth, reckoning each case at \$ 475.37½?

54. What would be the cost of 336 yards of carpeting at \$ 2.62½ per yard? Ans. \$ 882.

55. How much would 1250 cords of wood cost at \$ 8.75 per cord? Ans. \$ 10937.50.

177. To exchange or barter goods.

56. How many pounds of sugar, at 20 cents a pound, shall I give for 50 bushels of corn, at 80 cents a bushel?

OPERATION.

$$\frac{\overset{4}{\cancel{80}} \times 50}{\cancel{20}} = 200, \text{ Ans.}$$

This example is best solved by cancelling as in the margin. It may also be *analyzed* as follows: 50 bushels at 80 cents are worth 50 times 80 c. = 4000c., and 20c. in 4000c., 200 times, the number of pounds of sugar required, Ans. 200.

57. How many cords of wood, at \$ 8 per cord, will pay for 6 tons of hay, at \$ 20 per ton? Ans. 15.

58. How many tons of coal, at \$ 12.50 per ton, will pay for 16 yards of cloth, at \$ 6.25 per yard? Ans. 8.

59. How much flour at \$ 10.50 per barrel can be obtained for 150 bushels of potatoes at 75c. per bushel?

60. How many entire yards of broadcloth at \$ 5.75 per yard, can be bought for 3½ cords of wood at \$ 6.75 per cord, and what money will remain due? Ans. 4 yards, and 62½c. due.

61. How many bushels of wheat at \$ 3.50 per bushel would purchase 50 bushels of corn at \$1 50 per bushel? Ans. 21^{3}.

62. It requires 16 thousand shingles to cover the roof of a certain house, and they cost \$ 4.75 per thousand; how many days work at \$ 2.50 per day would it require to pay for them?

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177. Explain the operation in Ex. 56 by cancellation. How else may this example be solved?

63. How many yards of cloth, at $ 3.50 per yard, can be had for 3½ tons of hay, at $19.50 per ton?

BILLS.

178. A Bill of Goods is a written statement of articles sold, giving the price of each article and the cost of the whole.

An Account is a written statement of the items of debt and credit between two persons or companies.

The person or company who owes is the *Debtor*, and the one to whom something is due is the *Creditor*.

When a bill is paid it is usually *receipted* or signed by the creditor or by his authorized agent.

Receipts for an amount of $20 dollars or upwards, according to the laws of Congress, now require a revenue stamp to be affixed.

Find the cost of the several articles, and the amount or footing of each of the following bills.

(1.) *Boston, Sept.* 6, 1867.

Mr. John Low,

Bought of David Flint,

25 *lb. Sugar,*	*at*	16 *c.*
42 *lb. Butter,*	"	25 *c.*
15 *yd. Cloth,*	"	$ 3.33⅓

$ 64.50

Stamp

Received Payment,

David Flint.

---

178. What is a Bill of Goods? What an Account? Who is Debtor? Who Creditor? When should a bill be signed or receipted? By whom? What is said of affixing a revenue stamp?

(2.) *New York, Oct.* 15, 1867.

Messrs. SMITH & Co.,

Bought of ABEL ADAMS,

24 *gal. Molasses,*	*at*	87½*c.*
32 *gal. Syrup,*	"	$1.12½
48 *lb. Coffee,*	"	37½*c.*
16 *lb. Tea,*	"	62½*c.*

$85.

Stamp.

Received Payment,

ABEL ADAMS,
By L. SNOW.

(3.) *New Orleans, Dec.* 19, 1865.

Mr. JAMES FITCH,

1865. *To* HENRY DAY & Co., *Dr.*

June 4.	*To* 12 *Day's Algebras,*	*at*	87½*c.*
Aug. 9.	" 4 *Reams Paper,*	"	$2.75
16.	" 24 *Slates,*	"	37½*c.*
Nov. 11.	" 3 *Webster's Dictionaries,*	*at*	$8.75

$56.75

Stamp.

Received Payment,

JOHN SMITH,
For HENRY DAY & Co.

(4.) *Norwich, July* 5, 1867.

Mr. R. B. ALLEN,

Bought of JAMES ROBINSON & Co.

12 *pairs Men's Calf Boots,*	*at*	$4.75.
12 " " *Thick* "	"	3.75.
18 " *Boys'* " "	"	2.12½.

Stamp.

Received Payment,

JAMES ROBINSON & Co.

(5.) *Baltimore, Dec.* 15, 1866.

Messrs. John P. Jones & Co.,

1866.		*To* E. C. Johnson & Co., *Dr.*		
Mar. 4.	*To* 10 *tons Ice,*	*at*	$12.37½	
Apr. 8.	" 25 *bbl. Flour,*	"	9.25	
June 8.	" 100 *bush. Corn,*	"	87*c.*	
" "	" 50 *bush. Wheat,*	"	$1.75	
				$529.50
1866.		*Cr.*		
May 14.	*By Cash,*		$225.50	
June 8.	" *Merchandise,*		115.75	
Sept. 6.	" 5 *cords Oak Wood,*	*at*	9.75	
Dec. 1.	" 2 *tons Hay,*	"	15.45	
				$420.90
	Balance due E. C. J. & Co.			$108.60

Stamp.

Received Payment,

E. C. Johnson &Co.

Miscellaneous Examples in U. S. Money.

1. If 4 cords of wood cost $ 34.50, what is the price per cord? Ans. $ 8.62½.

2. What shall I pay for 7 tons of hay, at $ 16.75 per ton? Ans. $ 117.25.

3. My farm cost $ 3476.50 and my house cost $ 2347.75; how much more did I pay for the farm than for the house?

4. When beef costs 12½ cents per pound, what shall I pay for 1936 pounds? Ans. $ 242.

5. Bought 4 lb. tea at 75c., 6 yd. sheeting at 33⅓c., and 5yd. broad cloth at $ 3.25; what was the cost of all? Ans. $ 21.25.

6. If 8 yards of cloth cost $ 12, what will 12 yards cost?

7. If 16 barrels of flour cost $ 144, what will 12 barrels cost? Ans. $ 108.

8. If 4 tons of coal cost $ 35, what will 64 tons cost?

9. If 4 tons of hay cost $ 62.50, what will 48 tons cost?

10. Paid $ 4050 for 75 acres of land; at what price per acre shall I sell it to gain $ 225? Ans. $ 57.

11. Bought 22 pounds of sugar at 9 cents, 4 pounds of coffee at 56 cents, 3 pounds of tea at 75 cents, 5 gallons of molasses at 48 cents, and 5 barrels of flour at $ 9.75, and gave the merchant 6 ten-dollar bills, how much change shall he return to me? Ans. $ 2.38.

12. Bought 6 pounds of butter at 30 cents, 10 pounds of cheese at 18 cents, 24 pounds of rice at 6 cents, 7 pounds of raisins at 25 cents, 2 bushels of potatoes at 75 cents, 1 bushel of beans at $ 1.50, and 10 yards of sheeting, and gave 2 ten-dollar bills to the merchant, who returned $ 7.91; what was the price per yard of the sheeting? Ans. 23 cents.

13. A merchant bought 8 boxes of tea, containing 60 pounds each, for $ 312; but it being damaged he sold it at a loss of $ 72; at what price per pound did he sell it? How much did he lose on each pound?

14. A family, consisting of father, mother, and 2 children, desires to board by the sea during the summer, and can afford to pay $ 126; how many weeks can they remain, if the board of each parent costs $5.50, and of each child $3.50 per week? Ans. 7.

15. A laborer bought a bushel of potatoes for 75 cents, 6 pounds of sugar at 15 cents, a barrel of flour for $ 8.75, and 12 pounds of meat at 10 cents; he paid $ 5.35 in cash, and the balance in work at $ 1.25 per day; how many days did he work? Ans. 5.

16. A merchant found that for one year his whole profits were $ 8750; of this he paid $ 1250 for store rent, $ 2750 for clerk hire, and $ 1850 for other expenses; how much clear ...t remained?

17. A bookseller went to the city and bought a bill of books as follows: 12 readers at $ 1.25, 18 spellers at $ 37½, 10 geographies at $ 1.75, 8 primary geographies at $.62½. In payment he gave a hundred dollar bill, how much should he receive back?

18. It took 25 yards of carpeting at $ 1.87½ per yard for my sitting room, 50 yards of matting at $.65 per yard for my chambers, and 39 yards of oil-cloth at $ 1.25 per yard for my kitchen and halls; what was the amount of my bill?

Ans. $ 128.12½.

19. A ship carried to London from New Orleans 4500 bales of cotton, each bale weighing 475 pounds, at a freight of 2½ cents per pound, and other merchandise upon which the freight was $ 8595; what was the whole amount of the ship's freight?

20. A drover bought stock as follows: 7 horses, at an average price of $ 225; 50 sheep at $ 3.50; and one pair of oxen for $ 200; what did the whole cost him?

21. A gentleman found that his household expenses for one month were as follows: provisions $175.50, groceries $150.50, house rent $83.33; what was the amount? What would be the amount for one year?

22. A farmer sold the produce of his farm as follows: 150 bushels of potatoes at $.55, 175 bushels of corn at $ 1.25, and 50 bushels of wheat at $ 4.50 per bushel; what was the amount he received?

23. A builder took a contract to build a house; he paid for brick and stone work, with materials, $ 3500; for carpenter work, with materials, $ 2575.50; for painting $ 675, and for other work $1550; he received $10,000; how much were his profits?

COMPOUND NUMBERS.

ADDITION.

179. A COMPOUND NUMBER is composed of two or more denominations (Art. 92) which do not usually increase decimally from right to left; consequently, in adding the different denominations, we do not carry one for *ten, but for the number it takes of the particular denomination added, to make a unit of the next higher denomination;* thus, in adding Sterling or English money, we carry 1 for 4, 12, and 20, because 4qr. make 1d., 12d. make 1s., and 20s. make 1£.

Ex. 1. Add together 5£ 10s. 7d. 3qr., 6£ 18s. 11d. 2qr., 9£ 13s. 5d. 1qr., 17£ 16s. 9d. 3qr.

OPERATION.

£	s.	d.	qr.
5	10	7	3
6	18	11	2
9	13	5	1
17	16	9	3
39	19	10	1

We first arrange the numbers as in the margin. Then add the right-hand column as in simple numbers, and find the *amount* to be 9qr. = 2d. and 1qr. We write the 1qr. under the column of farthings, and add the 2d. to the column of pence; the amount of which we find to be 34d. = 2s. and 10d. We set the 10d. under the column of pence, and add the 2s. to the column of shillings, and find the amount to be 59s. = 2£ and 19s. We write the 19s. in the column of shillings, and add the 2£ to the column of pounds; the amount of which we find to be 39£, and the *whole* amount

£	s.	d.	qr.
39	19	10	1 Ans.

---

179. What is a compound number? How do they increase? What is said of carrying?

180. The *principle* of this process is precisely the same as in addition of simple numbers. Hence,

To add compound numbers,

Rule. *Write the numbers so that each denomination shall occupy a separate column, the lowest denomination at the right, and the others towards the left in the order of their values. Add the numbers in the lowest denomination, divide the amount by the number it takes of this denomination to make one of the next higher, set the remainder under the column, and carry the quotient to the next column. So proceed until all the columns are added.*

Proof. *The same as in Addition of Simple Numbers*

	2.				3.		
	£	s.	d.		gal.	qt.	pt.
	27	17	6		5	3	1
	19	15	10		4	2	1
	14	6	11		7	3	0
	28	19	9		4	0	1
Sum,	91	0	0		22	1	1
Proof,	91	0	0		22	1	1

Note. In writing, and also in adding the numbers of a SINGLE DENOMINATION, the rules of simple addition must be observed; thus in writing the pounds in Ex. 2, set units under units, tens under tens.

	4.					5.		
	lbs.	oz.	dwt.	grs.		A.	R.	rd.
	17	10	19	23		7	3	27
	13	7	13	19		2	2	31
	7	11	17	21		6	3	28
	27	10	15	20		9	3	39
Sum,	67	5	7	11		27	2	5
Proof,	67	5	7	11		27	2	5

6.

bush.	pk.	qt.	pt.
71	3	7	1
19	2	5	1
3	3	0	0
13	2	4	1

7.

t.	cwt.	qr.	lb.	oz.
7	19	3	20	13
5	14	2	16	14
3	17	3	23	2
4	16	1	19	8

8.

yd.	qr.	na.	in.
5	3	3	2
7	2	3	1½
9	3	2	2
7	3	3	2

9.

	rd.	yd.	ft.	in.
	7	4	2	10
	1	5	2	11
	6	3	1	7
	4	4	2	9
	21	2½	1	1
or	21	2	2	7

NOTE. A fraction occurring in the amount may sometimes be reduced to whole numbers of lower denominations; thus, in Ex. 9; we reduce the ½yd. to lower denominations = 1ft. 6in., this we add to the ft. and in. in the example, and have 21rd. 2yd. 2ft. 7in.

10. A trader bought 4 hhd. of sugar: the first weighed 10cwt. 3qr.. 17lb.; the second 13cwt. 1qr. 19lb.; the third 12cwt. 3qr. 18lb.; and the fourth 11cwt. 3qr. 27lb.; what did the whole weigh? Ans. 2t. 9cwt. 1qr. 6lb.

11. I have my winter's wood in four piles; in one are 4c. 5 c. ft. 12 cu. ft.; in another 2 c. 7 c. ft. 9 cu. ft.; in another 1 c. 6 c. ft. 13 cu. ft. and in the fourth 3 c. 5 c. ft. 11 cu. ft.; how much wood have I in all?

Ans. 13 c. 1 c. ft. 13 cu. ft.

12. A vintner has wine in 3 casks; in the first, 68gal. 3qt. 1pt. 3gi.; in the second, 79gal. 2qt. 1pt. 1gi.; in the third, 94gal. 3qt. 1pt. 3gi.; how much has he in the three casks?

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180. Rule for addition of compound numbers? Principle? Proof? Numbers of a single denomination, how written and added?

SUBTRACTION.

181. The principle is like that of subtraction of simple numbers. Hence,

To subtract compound numbers,

RULE. 1. *Write the less quantity under the greater, arranging the denominations as in addition.*

2. *Beginning at the right, take each denomination of the subtrahend from the number above it, and set the remainder beneath.*

3. *If any number of the subtrahend is greater than the number above it, add to the upper number as many as it takes of that denomination to make one of the next higher, and take the number in the subtrahend from the* SUM; *set down the remainder, and considering the number in the next denomination in the minuend* ONE LESS, *or that in the subtrahend* ONE GREATER, *proceed as before.*

PROOF. *As in subtraction of simple numbers.*

Ex. 1. From 12£. 9s. 6d. 3qr. take 8£. 7s. 9d. 2qr.

OPERATION.

	£	s.	d.	qr.
	12	9	6	3
	8	7	9	2
Sum,	4	1	9	1
Proof,	12	9	6	3

We take 2qr. from 3qr. and have 1qr. remaining, which we write under the qr. in the subtrahend. We see that we cannot take 9d. from 6d., we therefore borrow one from the 9 shillings, and reduce it to pence, which with the 6d. in the example = 18d. We now say 9d. from 18d. leave 9d. which we write in its proper place, under the pence in the example. Now, as one

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181. Rule for subtraction of compound numbers? Principle? Proof?

of the shillings has been borrowed, we say, 7d. from 8d., or what is practically the same, 8d. from 9d. leave 1d, and so proceed through the example.

2. From 8£ 5s. 7d. 1qr., take 3£ 12s. 4d. 3qr.
Ans. 4£ 13s. 2d. 2far.

3.

	t.	cwt.	qr.	lb.
Min.,	19	12	1	20
Sub.,	13	17	3	22
Rem.,	5	14	1	23

4.

lb.	oz.	dr.	sc.	grs.
13	5	3	1	10
7	9	1	2	17
5	8	1	1	13

5.

yd.	qr.	na.
9	1	2
3	3	3
5	1	3

6.

gal.	qt.	pt.
29	1	1
13	3	1

7.

lb.	oz.	dwt.	gr.
19	6	12	10
12	10	17	21

8.

mi.	fur.	rd.	yd.	ft.	
8	2	21	2	1	
3	7	33	3	2	
4	2	27	3½	2	
4	2	27	4	0	6 in.

NOTE. A fraction occurring in the answer, may, when reduced, contain a denomination higher or lower than any in the given example; as in Ex. 8, the ½yd. = 1ft. 6in. The 1ft. added to the 2ft. in the *first remainder* = 3ft. = 1yd. Add this to the 3yds. and we have 4mi. 2fur. 27rd. 4yd. 0ft. 6in.

Explain Ex. 1.

10.

deg.	mi.	fur.	rd.	yd.	ft.	in.
6	43	3	21	1	1	4
3	62	5	37	1	2	7

11. If I cut 15yd. 3qr. 2na. from a piece of cloth containing 31yd. 2qr., how much will remain?

Ans. 15yd. 2qr. 2na.

12. A grocer had a box of sugar containing 15cwt. 1qr. 13lbs. After taking out 9cwt. 3qr. 21lbs., how much remained in the box? Ans. 5cwt. 1qr. 17lbs.

13. An invoice of broadcloth, which cost 187£ 17s. 6d., was sold for 257£ 9s. 3d.; what was the gain?

Ans. 69£ 11s. 9d.

14. What is the difference in the longitude of two places, one 63° 30′ 15″ east, and the other 23° 45′ 30″ east?

182. To find the time between two dates.

Ex. 1. What is the difference of time between June, 11, 1856, and Oct. 4, 1859?

OPERATION.

Min.,	1859	10	4	
Sub.,	1856	6	11	
Rem.,	3 y.	3 mo.	23	Ans.

In subtracting an earlier from a later date, we call 30 days a month. We write first, the *number* of the year, month, and day of the *latest* date, and under it, the *number* of the year, month, and day of the *earliest* date, and subtract as in Art. 181, and the remainder will be the difference of time between the two dates.

2. Find the time from Sept. 23, 1862, to May 13, 1866.

Ans. 3y. 7mo. 20d.

3. Find the time from Aug. 17, 1858, to June 11, 1863.

Ans. 4y. 9mo. 24d.

4. Find the time from Feb. 8, 1856, to Aug. 1, 1860.
5. Find the time from March 7, 1857, to Nov. 20, 1865.

MULTIPLICATION.

183. In the multiplication of both simple and compound numbers, the *multiplier* is always a simple abstract number. The product is of the same kind as the multiplicand; for repeating a number does not change its nature.

The principle is the same as in multiplication of simple numbers. Hence,

To multiply a compound by a simple number we have the following

RULE. *Multiply the lowest denomination in the multiplicand, divide the product by the number it takes of that denomination to make one of the next higher, set down the remainder, add the quotient to the product of the next denomination, and so proceed till all the denominations are multiplied.*

PROOF. *Multiplication and Division of Compound Numbers prove each other.*

Ex. 1. Multiply 7£ 6s. 3d. 2qr. by 9.

OPERATION.

£	s.	d.	qr.	
7	6	3	2	Multiplicand.
			9	Multiplier.
65	16	7	2	Product.

We first say, 9 times 2qr. = 18qr. = 4d. and 2qr.; write the 2qr. under the farthings, and then say 9 times 3d. = 27d. and the 4d. added give 31d. = 2s. and 7d., and so proceed.

NOTE. As multiplication and division prove each other, it is profitable to teach the reverse operations simultaneously.

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182. How is the time between the two dates found? **183.** What kind of a number is the multiplier in all cases? What the product? The rule for multiplying a compound number? Proof? Explain Ex. 1.

2.

	rd.	yd.	ft.	in.
Multiply	9	4	2	7
By				8
Product,	78	5	2	8

3.

gal.	qts.	pt.	gi.
9	3	1	3
			7
69.	3	0	1

4.

lbs.	oz.	dwt.	grs.
6	7	13	17
			5
33	2	8	13

5.

lb.	oz.	dr.	sc.	grs.
3	9	6	2	14
				6

6.

yd.	qr.	na.	in.
7	2	3	2
			8

7.

°	′	″
17	30	45
		5

8.

gal.	qt.	pt.	gi.
9	2	1	3
			3

9.

bush.	pk.	qt.	pt.
9	3	6	1
			11

10. How much vinegar in 6 casks, each holding 37gal. 1qt. 1pt. 3gi. each? Ans. 224gal. 3qt. 0pt. 2gi.

11. What will be the weight of 3 loads of coal, if one load weighs 1 ton. 3cwt. 3qr. 27lbs. (long ton)?

12. What is the entire produce of a field of 8 acres, if one acre produce 38bu. 3pk. 5qts.? Ans. 311bu. 1pk.

13. If the moon's daily motion through the heavens is 33° 10′ 35″, how much of her orbit does she traverse in 17 days?

14. If a horse travel 42mi. 3fur. 37rd. in one day, how far will he travel in 16 days?

DIVISION.

184. Here, as in the three preceding sections, the principle is the same as in the corresponding operation in simple numbers. Hence,

To divide a compound number we have the following

Rule. *Divide the highest denomination of the dividend, and set down the quotient; if there is a remainder, reduce it to the next lower denomination; to the result add the given number of that denomination, and divide as before, setting down the quotient and reducing the remainder, and so proceed till all the denominations are divided.*

Proof. *Division is proved by multiplication.*

Ex. 1. Divide 27£ 15s. 6d. 3qr. by 8.

OPERATION.

	£	s.	d.	qr.
8)	27	15	6	3
	3	9	5	$1\frac{3}{8}$

We first divide 27£ by 8 and have a quotient of 3, and 3£ remaining. 3£ reduced to shillings, with the 15 shillings in the example, give 75 shillings, which divided by 8, give 9 and a remainder of 3s. This we reduce to pence, and add the 6d. in the example, and have 42d., which we divide as before, etc.

2. Divide 37 tons, 15cwt. 1qr. 21lb. by 12.
Ans. 3 tons, 2cwt. 3qr. 20½lb.

3. Divide 35y. 3mo. 17da. 13h. by 3.

4. Divide 76A. 2R. 25rd. by 5. Ans. 15A. 1R. 13rd.

5. If 5 loads of wood contain 9c. 7c. ft. 10cu. ft., what are the contents of 1 load?

6. How far will a man travel in one day, if he travel 171mi. 1fur. 29rd. in 7 days? Ans. 24mi. 3fur. 27rd.

7. If it take 250yd. 3qr. 2na. of carpeting to carpet nine

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14. Rule for dividing a compound number? Principle? Proof?

rooms, how many yards will it take to carpet one of the floors, they being of equal size? Ans. 27yd. 3qr. 2na.

8. If 4doz. spoons weigh 6lbs. 10oz. 16dwt., what will one dozen weigh? Ans. 1lb. 8oz. 14dwt.

9. A farmer put his wheat, consisting of 359bu. 3pk. 2qt., into 12 boxes of equal size; how much did each box contain?
Ans. 29bu. 3pk. 7qt. 1pt.

10. If 24hhd. of sugar weigh 4t. 14cwt. 3qr. 5lb., what is the weight of 1hhd.? Ans. 3cwt. 3qr. 20lb.

11. A farmer divided his farm consisting of 446A. 3R. 30 rd., equally among his 8 children; what was the share of each?

MISCELLANEOUS EXAMPLES.

1. A blacksmith bought 5cwt. 2qr. 21lb. of iron at one time, 1t. 11cwt. 18lb. at another; how much did he buy in all?
Ans. 1t. 16cwt. 3qr. 14lb.

2. How many *pounds* of iron did the above blacksmith buy, and what did it cost him at 5 cents per pound?

3. A farmer raised in one field 302bu. 2pk. 7qt. of oats, in another 290bu. 3pk. 4 qt.; how much more did he raise in one than the other? Ans, 11bu. 3 pk. 3 qt.

4. I have a piece of land containing 50A.; if I sell 25A. 3R. 25rd. of it, how much shall I have left?

5. If a ship sail 2° 2′ 30″ in one day, how far will she sail in a week? Ans. 14° 17′ 30″.

6. How much wood in 5 loads, each containing 1c. 3c. ft. 16cu. ft.?

7. What would be the crop of hay on 10A. if the product of 1A. was 3t. 10 cwt. 2 qr.? Ans. 35 t. 5 cwt.

8. Divide 2t. 7 cwt. 2qr. 10 lb. by 7.

9. If 1 cubic yard of stone weigh 2 tons, 7cwt. 25lb., what is the weight of 1 cubic foot? Ans. 175lb.

PERCENTAGE.

185. The term PER CENT means by the hundred; thus, by *five* per cent of a ton of coal, we mean five one hundredths of it; i. e. five parts out of every one hundred parts; 6 per cent of a sum of money, is six one-hundredths of the sum, i. e. $ 6 out of every $100.

NOTE. Instead of the words *per cent* it is quite customary in writing to use the sign %; thus, 6 per cent is written 6 %; 4½ per cent 4½ %.

186. The RATE PER CENT is the *number for each hundred;* thus, 6 % is $\frac{6}{100}$, or .06, i. e. 6 parts for each hundred parts.

187. The PERCENTAGE is the sum computed on the given number; thus, the percentage on $ 200 at 6 per cent is $ 12.

NOTE. The pupil should be cautioned not to confound *per cent* and *percentage*. The distinction should clearly be borne in mind.

188. The BASE of percentage is the number on which the percentage is computed; thus, we say the percentage on $ 500, at 8 per cent is $ 40. Here $ 500 is the BASE, 8 is the PER CENT and $ 40 is the PERCENTAGE; also, 10 per cent. of 2000 lb. (a ton) of coal is 200lb.; here 2000 lb. is the *base,* 10 is the *per cent* and 200lb. is the *percentage.*

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185. Meaning of *per cent?* **186.** Rate per cent? Illustrate. **187.** Percentage? Illustrate. **188.** Base of percentage? Explain the three last mentioned terms by an example.

189. The rate per cent being a certain number of hundredths, may be expressed either *decimally*, or by a *common fraction*, as in the following

TABLE.

			Decimals.		Common Fractions.
1	per cent	is	.01	=	$\frac{1}{100}$.
2	per cent		.02	=	$\frac{1}{50}$.
5	per cent		.05	=	$\frac{1}{20}$.
$6\frac{1}{4}$	per cent		.0625	=	$\frac{1}{16}$.
$8\frac{1}{3}$	per cent		$.08\frac{1}{3}$	=	$\frac{1}{12}$.
10	per cent		.10	=	$\frac{1}{10}$.
$12\frac{1}{2}$	per cent		.125	=	$\frac{1}{8}$.
$16\frac{2}{3}$	per cent		$.16\frac{2}{3}$	=	$\frac{1}{6}$.
$18\frac{3}{4}$	per cent		.1875	=	$\frac{3}{16}$.
20	per cent		.20	=	$\frac{1}{5}$.
25	per cent		.25	=	$\frac{1}{4}$.
$33\frac{1}{3}$	per cent		$.33\frac{1}{3}$	=	$\frac{1}{3}$.
50	per cent		.50	=	$\frac{1}{2}$.
	etc.			etc.	

NOTE. When the per cent is expressed by a decimal of more than 2 places, the figures after the second decimal place must be regarded as parts of 1 per cent; thus, (in the seventh line of the foregoing table,) .125 is $12\frac{5}{10}$ or $12\frac{1}{2}$ per cent.

Ex. 1. Write the decimal for 6 per cent. Ans. .06.

2. Write the decimal for 4 per cent; 12 per cent; 8 per cent; 15 per cent; 25 per cent; $16\frac{2}{3}$ per cent.

3. Write the common fraction for 5 per cent; 10 per cent; $12\frac{1}{2}$ per cent; $6\frac{1}{4}$ per cent; $33\frac{1}{3}$ per cent.

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189. In what ways may the rate be expressed? If expressed decimally by more than two figures, what are the figures after the second decimal place?

Note. Too much pains can not be taken to make the pupil thorough in exercises like those in the last two examples.

Case 1.

190. To find the percentage, the base and rate per cent being given.

Ex. 1. John Dow had \$ 360, but lost 5 per cent of it, how many dollars did he lose?

$$\begin{array}{r} \$\,360 \\ .05 \\ \hline \$\,18.00 \end{array}$$

Since 5 per cent is $.05 = \frac{1}{20}$, we find the loss, (percentage), by multiplying \$360 by .05 or by $\frac{1}{20}$. Hence, the

Rule. *Multiply the base by the rate per cent expressed decimally or as a common fraction, and the product will be the percentage.*

2. What is 20 per cent of \$ 160.? Ans. \$ 32.

3. The base is 560 and the rate per cent 40; what is the percentage?

$$560 \times .40 = 224, \text{ Ans.}$$

$$\text{Or, } 560 \times \tfrac{2}{5} = 224, \text{ Ans.}$$

4. What is 16⅔ per cent of 180 barrels of flour? Ans. 30 bbl.

5. What is 5 % of \$ 200? Ans. \$ 10.

6. What is 8⅓ % of 240 tons of coal? Ans. 20 tons.

7. In a certain school there are 720 pupils, 33⅓ per cent are more than 12 years of age; how many are over 12 years old? Ans. 240.

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190. Rule for finding the percentage when the base and rate are given?

8. A flour merchant bought 1200 bbls. of flour, but 8 per cent of it was injured by rain; how much was injured?

9. A pupil had a lesson of 40 words, but failed on 10 per cent of them; on how many words did he fail?

10. A city containing 35000 inhabitants, had 15 per cent of the number in school children; how many school children were there? Ans. 5250.

11. A merchant fails in business, owing $ 12600, and can pay but 35 per cent of his debts; how much will his creditors lose?

$$100 - 35 = 65$$

$$12600 \times .65 = \$8190, \text{ Ans.}$$

12. Bought 600 boxes of oranges, but on opening them, I find 8 % of them spoiled; how many were lost? Ans. 48.

13. A gentleman sold his house for $ 6500. $ 600 he received in cash and took a note for the balance; how much cash did he receive?

14. I have $ 1580 on deposit in the bank. If I draw out 12½ per cent of it, what per cent will remain? What amount of money will remain? Last Ans. $1382.50.

15. In an orchard consisting of 1200 trees, 30 % bear apples, 45 % bear pears, and the remainder bear peaches; how many bear peaches? Ans. 300 trees.

CASE 2.

191. To find the rate per cent when the base and percentage are given.

Ex. 1. What per cent of $ 24 is $ 6.?

$\frac{6}{24} = \frac{1}{4} = .25$, Ans.

$ 6 is ¼ of $ 24 and ¼ reduced to a decimal = .25 i. e. $ 6 = 25 per cent of $ 24.

Hence the

Rule. *Make the percentage the numerator of a common fraction and the base the denominator, and then reduce this fraction to a decimal.*

2. What per cent of $ 20 is $ 5? Ans. .25.

3. What per cent of $ 400 is $ 50? Ans. .12½.

4. A man having $ 6000, paid $ 1200 for a piece of land; what per cent of his money did he expend? Ans. .20.

5. My salary is $ 1800 per annum, and my expenses $ 1600; what per cent of my income do I spend? What per cent save?

6. Bought a cask of vinegar containing 84 gallons; 21 gallons have leaked out; what per cent have I lost?

7. Purchased a horse for $ 160, and sold him for $ 128, what per cent did I lose?

Case 3.

192. To find the base when the percentage and the rate are given.

Ex. 1. $ 12 is 4 per cent of what sum? Ans. $ 300.

If $ 12 is 4 per cent, 1 per cent will be ¼ of $ 12 which is $ 3, and if $ 3 is 1 per cent, 100 per cent will be 100 times $ 3 = $ 300. The same result is obtained by multiplying by 100 *first,* and then dividing by 4; thus, 1200 ÷ 4 = 300. Hence,

Rule. *Multiply the percentage by* 100, *and divide the product by the rate, and the quotient will be the base.*

2. $ 12.60 is 6 % of what sum? Ans. $ 210.

3. $ 15 is 8 % of what sum? Ans. $ 187.50.

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191. Rule for finding the rate per cent when the base and percentage are given? **192.** Rule for finding the base when the percentage and rate are given?

4. $ 36.30 is 3 % of what sum?

5. $ 75 is 6 % of what sum? Ans. $ 1250.

6. $ 12 is 4 % of what sum? Ans. $ 300.

7. A gentleman purchased a farm for $ 6900, which was 20 per cent of his entire property. What was he worth?

8. James Marvin cures fish for Thomas Tarlton, receiving in pay 12½ per cent of the quantity cured. His share this season is 4680lbs; what quantity did he cure?

9. A merchant saves $ 3000 annually, which is 16⅔ per cent of his entire receipts; what are his receipts?
Ans. $ 18000.

10. A farmer sold 56 sheep, which was 12½ per cent of his whole flock. How many sheep had he in all? Ans. 448.

INTEREST.

193. INTEREST is money paid *for the use* of money.

The PRINCIPAL is the sum for which interest is paid.

The AMOUNT is the *sum* of the *principal* and *interest.*

194. An example in interest is only a question in *percentage.* The *principal* is the *base* of percentage, the *interest* is the *percentage,* and the interest on $1 for a year is the *rate* written decimally.

195. The *rate* is usually *fixed by law,* and a higher rate than the law allows is called *usury.*

In New England and most of the United States the *legal* or *lawful* rate is 6 per cent; in New York, 7 per cent.

NOTE. In Massachusetts a higher rate than six per cent may legally be agreed upon. In this book 6 per cent is understood when no per cent is mentioned.

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193. What is Interest? What is the Principal? What is the Amount? **194.** What is said of an example in Interest? Explain how the latter is like one in percentage. **195.** What is the Rate? The legal Rate?

196. To find the interest on any sum at 6 per cent for a given time.

Ex. 1. What is the interest of $240, for one year 6 months?

OPERATION.

$240. Principal.
2) 2.40, Interest for 2 mo.
1.20, Interest for 1 mo.
18 Months = 1y. 6mo.
960
120
$21.60, Interest for 18 mo.

ANALYSIS.

Any sum of money at 6 % will gain $\frac{1}{100}$ of itself in 2 months. We find this by removing the decimal point two places to the left, that is, by dividing by one hundred. We then divide this interest by 2 which gives the interest for one month. This, we multiply by the number expressing the given time in months, and have the interest of $240 for 1y. 6mo. = $21.60.

Ex. 2. What is the interest of $244.40 for 1 year 4 mo. 12 days?

OPERATION.

$244.40, Principal.
2) 2.444 = Interest for 2 mo.
1.222 = Interest for 1 mo.
16.4 mo. = 1y. 4mo. 12d.
4888
7332
1222
$20.0408 = Int. for $16\frac{4}{10}$ mo.

ANALYSIS.

In this example we proceed as in Ex. 1, till we come to the days. 12 days = $\frac{12}{30}$ of a month, which we reduce to a decimal = .4 and annex to the 16 months, and multiply as before.

Hence to find the interest on any sum for any time at 6 per cent we have the following

RULE. *Remove the decimal point in the principal, two places to the left; divide this result by two and multiply this*

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96. How do you find the interest on any sum for any time at 6 per cent? lain the steps.

quotient by the time in months and tenths of a month (the days divided by 3 = *tenths of a month), and the product will be the interest at* 6 *per cent for the given time.*

Ex. 3. What is the interest of $ 46 for 1 month?

$46. ÷ 100 = 46, and .46 ÷ 2 = $ .23, Ans.

4. What is the interest of $ 246.58 for 1 month?

$ 246.58 ÷ 100 = 2.4658, ÷ 2 = $ 1.2329.

5. What is the interest of $ 4, for 1 month?

$ 4 ÷ 100 = .04, ÷ 2 = .02, Ans.

6. What is the interest of $ 1, for 1 month?

$ 1. ÷ 100 = .01, ÷ 2 = .005, Ans.

7. What is the interest of $ 56.298, for 1 month?

8. What is the interest of $ 864.25, for 1 month?

Ans. $ 4.32125.

9. What is the interest of $ 69.42, for 1 month?

10. What is the interest of $ 2468.20, for 1 month?

11. What is the interest of $ 59.278, for 1 month?

12. By what will you multiply the interest for one month to find the interest for 1 year, 4 months?

1y. 4mo. = 16mo. Ans. 16.

13. By what will you multiply to find the interest for 2 years, 7 months?

14. By what will you multiply to find the interest for 15 days? $15\text{d.} = \frac{15}{30}\text{mo.} = \frac{1}{2}\text{mo.} = .5\text{mo.}$ Ans. .5.

15. By what will you multiply to find the interest for 21 days?

16. By what will you multiply to find the interest for 22 days? $22\text{d.} = \frac{22}{30}\text{mo.} = .7\frac{1}{3}\text{mo.}$ Ans. $.7\frac{1}{3}$.

17. By what will you multiply to find the interest for 9 months, 13 days? $9\text{mo. } 13\text{d.} = 9\frac{13}{30}\text{mo.} = 9.4\frac{1}{3}\text{mo.}$

Ans. $9.4\frac{1}{3}$.

18. By what will you multiply to find the interest for 1y. 7mo. 19d.?

19. By what will you multiply to find the interest for 2y. 9mo. 20d.?

20. What is the interest of $164.30, for 18 days?

$164.30 \div 100 = 1.6430, \div 2 = .8215, \times .6 = \$.4929.$

21. What is the interest of $58.64, for 2y. 3mo. 21d.?

$\$58.64 \div$ by $100 = .5864, \div 2 = .2932, \times 27.7 = \$8.12 +.$

22. Find the interest on $649.28, for 3y. 4mo. 26 days.
Ans. $132 66.

23. Find the interest on $384.92, for 2y. 5mo. 27 days.

24. Find the interest on $87.25, for 1y. 1mo. 10 days.
Ans. $5.816.

25. What is the amount of $142.80, for 1y. 6mo. 24 days?
Ans. $156.223.

Note. The amount is the *sum* of the principal and interest added together.

26. What is the amount of $234.60, for 6y. 7mo. 8 days?
Ans. $327.57+.

27. What is the amount of $104.20, for 6mo. 19 days?
Ans. $107,655+.

28. What will $380.50 amount to in 1y. 5mo. 10 days?

29. Find the interest of $60, for 60 days. Ans. $.60.

30. Find the interest of $30, for 90 days.

31. Find the interest of $240.60, for 2y. 11mo. 28d.
Ans. $43.227.

32. Find the amount of $350, for 3y. 7mo. 10d.

33. Find the interest on $3, for 7 days. Ans. .0035.

34. Find the interest on $.80 for 10 days. Ans. .0013⅓.

35. Find the interest on $2.42, for 25 days.

36. Find the amount of $ 5, for 12 days. Ans. $ 5.01.

37. Find the amount of $ 75.60, for 8mo. 29d. Ans. $78.989.

38. Find the amount of $ 3000, for 2y. 8mo.

39. Find the amount of $ 230, for 7y. 6mo. Ans. $ 333.50.

40. Find the interest of $ 394.27, for 8y. 23d.

41. Find the amount of $ 6000, for 9y. 7mo. 23d.

197. In all the previous examples, the interest has been computed on the basis of 6 per cent. In Ex. 42, we first cast the interest at 6 per cent, as before, and find it to be $17.001; this we divide by 6, which gives the interest at 1 per cent; and lastly, we multiply this interest by 7 and have the interest at 7 per cent = $ 19.83+.

42. What is the interest of $ 164, for 1y. 8mo. 22d. at 7 per cent? Ans. $ 19.83 +.

43. What is the interest of $ 270.60, for 3y. 11mo. 19d. at 8 per cent? Ans. $ 85.93.

44. What is the interest of $ 492.75, for 2y. 7mo. at 7½ per cent?

45. Find the interest on $ 75.87, for 5y. 3mo. at 9 per cent. Ans. $ 35.848.

46. Find the interest on $ 894.20, for 3y. 6mo. 15d. at 8 per cent. Ans. $ 253.35.+

47. Find the amount of $ 382.85, for 4y. 10mo. at 9 per cent.

48. Find the amount of $ 69.47, for 3y. 8mo. at 6½ per cent. Ans. $ 86.02.

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197. How is interest found for any other rate than 6 per cent? Explain the steps. How is the difference of time between two dates found?

49. Find the interest of $ 609.42, for 8y. 7mo. 6d. at 4 per cent. Ans. $ 209.64.

50. Find the interest on $ 493.85, for 2y. 3mo. at 3½ per cent. Ans. $ 38.89.

51. Find the interest on $ 6000, for 8y. 7mo. 17d. at 5½ per cent.

52. What is the interest of $ 64.82 from June 24, 1856, to Oct. 9, 1859? Ans. $ 12.80.

NOTE. First find the difference of time = 3y. 3m. 15d.

53. What is the interest of $ 85.93 from Jan. 6, 1850, to June 1, 1854?

54. What is the interest of $ 942.87 from Aug 13, 1861, to Nov. 7, 1864? Ans. $ 182.91.

55. What is the interest of $ 293.80 from Feb. 19, 1860, to Sept. 4, 1863? Ans. $ 62.43.

56. What will $ 843.92 amount to, from Aug 28, 1862, to Jan. 1, 1866? Ans. $ 1013.12.

57. What is the interest of $ 59.75 from Dec. 29, 1858, to June 7, 1861, at 7½ per cent? Ans. $ 10.92.

58. What will be the amount of $ 642.90 from July 4, 1862, to the day of Gen. Lee's surrender?

59. Find the interest of $ 8942, from the fall of Fort Sumpter to the evacuation of Richmond?

PROFIT AND LOSS.

198. PROFIT AND LOSS, are commercial terms, used to indicate the gain or loss in buying and selling goods, and in business transactions generally.

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98. What is Profit and Loss?

CASE 1.

199. To find the absolute gain or loss on a quantity of goods sold at retail, the purchase price of the whole quantity being given.

Ex. 1. Bought 200lb. of coffee for $ 50 and sold it at $ .40 per lb.; how much did I gain on the whole?

$ 200. × .40 = $ 80.; 80 — 50. = 30. Ans. $ 30.

Hence the

RULE. *Find the whole sum received for the goods, and the difference between this and the purchase price will be the gain or loss.*

2. Bought 32 yards of cloth for $ 48 and sold it at $ 2.20 per yard; what was my whole gain? Ans. $ 22.40.

3 Bought 4cwt. 3qr. 16lb. sugar for $ 61.375, and sold it at $.20 per lb. How much did I make on the whole?

4. Bought 960 oranges for $ 19.20 and sold them for 5 cents apiece; what was my entire gain?

5. Bought 15 doz. pencils at $ 1.20 per doz. and sold them at $.15 each; what did I make?

CASE 2.

200. To find the per cent of gain or loss when the cost and selling price are given.

Ex. 1. Bought 12 yards of cloth for $ 18. and sold it at $ 2.50 per yd. What was my whole gain, and what my gain per cent?

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199. Rule for finding absolute gain or loss?

200. Rule for finding the per cent of gain or loss on the cost price?

$ 2.50 Selling price.
12 yards.
30.000 = Sum rec'd.
18 = Cost.
$12 = Whole gain.
$\frac{12}{18} = \frac{2}{3} = .66\frac{2}{3}$.

We first find the whole gain to be $ 12 which is $\frac{12}{18} = \frac{2}{3}$ of the cost. This we reduce to a decimal, and have $.66\frac{2}{3}$; i. e., the gain $ 12, is $66\frac{2}{3}$ per cent of the cost $ 18. Hence, the following.

RULE. *Having found the total gain or loss by Case 1, make a common fraction by writing the gain or loss for the numerator and the cost of the article for the denominator, and then reduce this fraction to a decimal.*

2. Bought a gross of steel pens for $ 1.20, and sold them at 1 cent apiece. What was my gain per cent?

Ans. 20%.

3. I purchased 1 bushel of cherries for $ 3, and sold them at 15 cents per quart. What was my gain per cent?

4. Sold a horse for $ 100 which cost me $ 150. What was my loss per cent?

CASE 3.

201. To find the selling price, the cost and gain or loss per cent being given.

Ex. 1. Bought cloth at $ 6 per yard and wish to sell it at an advance of 20%. What is my selling price?

$ 6.00
1.20
$ 7.20000, Ans.

I shall sell what cost me $ 1. for $ 1.20 and what cost me $ 6 for six times $ 1.20 = 7.20.

Ex. 2. Bought goods for $ 300, but they being damaged I am willing to sacrifice 15 % of their value; for what shall I sell them?

---

201. Rule to find the selling price, the cost and gain or loss per cent being given?

$$\begin{array}{r} \$300. \\ .85 \\ \hline 1500 \\ 2400 \\ \hline \$255.00 \end{array}$$

If I lose 15%, I shall sell what cost me $ 1. for $.85, and shall sell what cost me $ 300. for 300 times $.85 = $ 255.

Hence, we have the following

RULE. *Multiply the* COST *by* 1 *minus the* LOSS *or plus the* GAIN PER CENT, *and the product will be the selling price.*

3. For what must I sell sugar that cost me 15 cents per pound, to gain 30% ? Ans. $ .19½.

4. Bought a pair of skates for $ 4.00; for how much must I sell them to gain 10% ?

CASE 4.

202. To find the first cost of an article, the selling price and gain or loss per cent being given.

Ex. 1. Sold wine at $6 per gal. and by so doing I made 20 per cent on the cost; what was the cost?

$$\tfrac{100}{120} = \tfrac{5}{6}$$

$$\$6. \times \tfrac{5}{6} = \$5. = \text{cost.}$$

That which cost $ 1. sold for $ 1.20, therefore, the cost was $\frac{100}{120} = \frac{5}{6}$ of the selling price; hence, the cost was $\frac{5}{6}$ of $ 6. = $ 5.

Hence we have the following

RULE. *Make a fraction by writing* 100 *for the numerator and* 100 *minus the loss or plus the gain per cent for a denominator, then multiply the selling price by this fraction.*

2. Flour selling at $ 15 per bbl., yields a profit of 25%; what is the cost? Ans. $ 12.

3. Sold 8 yards of cassimere at $ 3 per yd., and made 20% by the sale; what was the cost? Ans. $2.50 per yard.

4. Bought a quantity of wheat, but it being damaged, I sell it at $ 1.50 per bushel, and by so doing lose 25 per cent on the cost; what was the cost? Ans. $ 2.00.

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202. Rule for finding the cost, the selling price and gain or loss being given?

MISCELLANEOUS EXAMPLES.

1. Add 120×2 to $972 \div 36$. Ans. 267.

2. The difference between two numbers is 16, and the larger is 92; what is the smaller?

3. What is the difference between $2446352 + 694701$ and $2146705 - 839241$?

4. What is the difference between 246001×1641 and 245897×321? Ans. 324754704.

5. Multiply 12948 by 287, subtract 58672, and divide the remainder by 218.

6. How many cords in a pile of wood 86 ft. long, 4 ft. wide, and 9 ft. high? Ans. $24\frac{3}{16}$.

7. Divide $\frac{4}{9}$ of $\frac{3}{8}$ by $\frac{5}{7}$ of $\frac{14}{20}$. Ans. $\frac{1}{3}$.

8. What number multiplied by $8\frac{1}{5}$ gives 205?

9. What number divided by $19\frac{2}{3}$ gives 36? Ans. 708.

10. At \$$10\frac{5}{8}$ per ton, what will be the cost of $\frac{3}{4}$ of a ton of coal?

11. If $1\frac{3}{4}$ yards of cloth are required for 1 coat, how many coats may be made from $22\frac{6}{8}$ yards? Ans. 13.

12. What is the difference between eight hundred thousand and eight hundred-thousandths? Ans. 709999.99992.

13. If 18 gentlemen have \$646.70 each, what sum have they all? Ans. \$ 11640.60.

14. Bought 19 bbls. of flour for \$261.25; what was the price per bbl.?

15. If the crop of hay on 1 acre is 1 ton, 18 cwt. 3 qr. 15 lbs. what will be the crop on 10 acres? Ans. 19 t. 9 cwt.

16. The population of a certain city is 29460; what will it be a year hence if it gains 5%?

17. If a farmer raises 2650 bushels of wheat one year and 2968 the next; what per cent did his crop increase?
Ans. 12 per cent.

18. A man with a salary of $1600, spends $1200; what per cent of his salary did he save?

19. What is the interest of $1.00 for 8 yrs. 8 mos. and 6 days? Ans. $0.521.

20. What is the interest of $500.00 for 60 days, at $7\frac{3}{10}$%?

21. Find the interest of $460 for 2 yrs. 4 mos. and 18 days.
Ans. $65.78.

22. What is the interest of $84.75 for 10 yrs. 6 mos. 24 days, at 9%? Ans. $80.597.

23. The credit side of an account is composed of the following items: $1500.75, $655.30, $175.875; what is the whole amount?

24. The debit side of the above account has the following items: $576.37, $1025.50, and $1850.00; what is the whole amount?

25. On which side of the above account is the balance, and how much is it? Ans. debit; $1119.945.

26. How many yards of cloth at $2.25 per yard should be received for 8 cords of wood at $6.20 per cord?
Ans. $22\frac{2}{25}$ yds.

27. In 1 week, 1 day, 16 hours, and 40 min. how many seconds? Ans. 751200.

28. Resolve 1820 into its prime factors.

29. Add $\frac{2}{9}$, $\frac{5}{6}$, $\frac{1}{4}$, and $\frac{7}{8}$. Ans. $2\frac{13}{72}$,

30. Bought a house-lot 120 ft. long and 90 ft. wide, at 10 cts. per square foot. The cost of the lot was 12% of the cost of the house; what was the cost of both?
Ans. $10080.00.

31. If 12 bbls. flour cost $135.00, what will 50 bbls. cost?

32. Divide four thousand by eight thousandths.

33. If $\frac{3}{13}$ of a vessel cost $3690, what will $\frac{2}{5}$ cost? Ans. $6396.

34. What is the interest of $1245.60 from Jan. 20, 1866, to May 2, 1867?

35. The population of the New England States, in round numbers, is as follows: Maine 628000, New Hampshire 326000, Vermont 315000, Massachusetts 1231000, Rhode Island 175000, Connecticut 460000; what is the entire population of New England?

36. Bought corn at $2.00 a bag, and sold it for $1.60; how much did I lose per cent? Ans. 20%.

37. If 1 ton of coal costs $7.75, how many tons can be bought for $147.25?

38. Bought a pile of wood 40 ft. long, 12 ft wide, and 15 ft. high, at $6.00 a cord; how many cords were there, and what was the expense? Ans. $56\frac{1}{4}$ cords, and $337.50.

39. Find the greatest common divisor of 1504 and 3478.

40. Find the least common multiple of 6, 12, 18, 36, and 54.

41. If it cost $25.00 for 1000 miles travel, what is that a mile? Ans. $2\frac{1}{2}$ cents.

42. Bought a house for $1800.00, which was $\frac{3}{7}$ of what I paid for my farm; what was the cost of the farm?
Ans. $4200.00.

43. A manufacturer sends 34 cases of shoes to a commission merchant. These are sold at $140 a case. The commission is $2\frac{1}{2}$ per cent. What is the whole amount of the commission received, and how much is due the manufacturer?

44. Multiply 25893 by .000402. Ans. 10.408986.

45. If I give .35 of a cord of wood for 1 day's work, how much should I give for 64.50 days' work? How much would it be worth at $5.00 a cord?

46. What is the interest of $800.50 from June 19, 1860, to Nov. 4, 1865? If the above interest is payable in gold, at a premium of 40%, what would be the current value?

47. Sold a horse and carriage and lost $50 by the transaction. This was 10% of the cost. What was the cost and selling price? Ans. $500.00, cost; $450.00, selling price.

48. If $200 gain $24 in 2 years, what will $50 gain in the same time?

49. Bought 15 shares of bank stock for $1650, and sold them at an advance of $5.50 a share. What per cent did I gain? Ans. 5%.

50. Bought 160 acres of land for $15 an acre. Sold 10 house-lots at $50 each, a quantity of lumber and wood for $1725, and the remainder of the land for $1550. What did I gain?

THE METRIC SYSTEM

OF WEIGHTS AND MEASURES.

203. In the Metric System the Scales are all decimal as in United States Money. It is so named from the Meter, which is one ten-millionth of a quadrant, or one forty-millionth of the circumference of the earth measured over the poles.

LONG MEASURE.

204. The principal unit of length is the *Meter*, which is 39.37 inches long.

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203. What is said of the scales of the Metric System? Why is this System so called?

204. What is the principal unit of Long Measure? What is its length?

TABLE.

10	*Millimeters* (^{mm})	make	1 Centimeter.
10	Centimeters	"	1 Decimeter.
10	Decimeters	"	1 METER (^{m}).
10	Meters	"	1 Dekameter.
10	Dekameters	"	1 Hectometer.
10	Hectometers	"	1 *Kilometer* (^{km}).

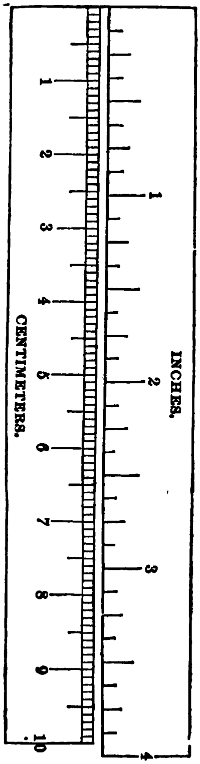
Km.	Hectom.	Dekam.	Meters.	Decim.	Centim.	Mm.
					1 =	10
				1 =	10 =	100
			1 =	10 =	100 =	1,000
		1 =	10 =	100 =	1,000 =	10,000
	1 =	10 =	100 =	1,000 =	10,000 =	100,000
1 =	10 =	100 =	1,000 =	10,000 =	100,000 =	1,000,000

NOTE 1. About twenty-five (more exactly 25.4) millimeters make one inch. The meter is about *three* feet, *three* inches, and *three*-eighths of an inch, which may be remembered as the rule of the *three threes.*

NOTE 2. The kilometer is the common unit for road measure, and is about two hundred rods, or five-eighths of a mile. Five meters make about one rod.

The accompanying scale exhibits one decimeter divided into ten centimeters, each centimeter being divided into ten millimeters. With it is a four-inch scale divided into eighths of an inch.

Give the Table of Long Measure. What is the Common Unit of Road Measure? Its length? Draw a section of the accompanying scale, and explain it.



One Decimeter, or $\frac{1}{10}$ of a Meter.

These measures, as well as all the other metric measures and weights, are written like whole numbers and decimals. Thus, 3 kilometers, 8 hectometers, 7 meters, and 5 decimeters, are written 3807.5^{m}. Large distances, as in road measure, are given as kilometers and decimals. Thus, 47.34^{km} stands for 4 myriameters, 7 kilometers, 3 hectometers, and 4 dekameters. Small distances are usually expressed in millimeters, or in centimeters.

The names of the several larger units of length are formed from the word *Meter*, by prefixing *Myria* for 10,000, *Kilo* for 1000, *Hecto* for 100, and *Deka* for 10. The smaller units are denoted by *Deci* for $\frac{1}{10}$, *Centi* for $\frac{1}{100}$, and *Milli* for $\frac{1}{1000}$. In the same way, as will be seen hereafter, are formed the names of weights and of measures of surface and capacity.

NOTE 1. The first series of prefixes is from the Greek, the second from the Latin language.

NOTE 2. The terms *Dime*, *Cent*, and *Mill*, in United States money, for the tenth, hundredth, and thousandth parts of a dollar, are analogous to the terms *Decimeter*, *Centimeter*, and *Millimeter*.

NOTE 3. The Metric System is used in France and many other countries, and is legalized in the United States and Great Britain.

How are these weights and measures written? Illustrate. What prefixes indicate the larger denominations? What the smaller?

205. To reduce a larger denomination in the Metric System to a smaller, or a smaller to a larger:

Multiply or divide by 10, 100, 1000, *&c., as the case may require.* (*Art.* 159.)

Ex. 1. Reduce 64 meters to millimeters.

$1^{m} = 1000^{mm}$ $64^{m} = 1000^{mm} \times 64 = 64000^{mm}$. Ans.

2. Reduce 8500 millimeters to meters. Ans. 8.5^{m}.

3. Reduce 95000 meters to kilometers. Ans. 95^{km}.

206. Metric measures and weights are added, subtracted, multiplied and divided like whole numbers and decimals.

Ex. 1. Add 4.5^{m}, 26.25^{m} and 9450^{mm}. Ans. 40.2^{m}.

2. From 978^{m} take 392.64^{m}. Ans. 585.36^{m}.

3. Multiply 736.45^{m} by 7.

4. Divide 1840.86^{m} by 63. Ans. 29.22^{m}.

SQUARE MEASURE.

207. The principal units of square measure are the *Are* and the *Square Meter*. The Are is a square whose side is 10 meters, and therefore contains 100 square meters.

TABLE.

100 Sq. Centimeters	make	1 Sq. Decimeter.
100 Sq. Decimeters	"	1 Centare, or sq. meter.
100 Centares, or sq. meters	"	1 Are (^{ar}).
100 Ares	"	1 Hectare (^{ha}).
100 Hectares	"	1 Sq. Kilometer.

Sq. Kilom.	Hectares.	Ares.	Sq. Meters, or Centares.	Sq. Decim.	Sq. Centim.
				1 =	100
			1 =	100 =	10,000
		1 =	100 =	10,000 =	1,000,000
	1 =	100 =	10,000 =	1,000,000 =	100,000,000
1 =	100 =	10,000 =	1,000,000 =	100,000,000 =	10,000,000.000

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205. How is Reduction performed? **206.** How are these measures added, subtracted, &c.? **207.** What are the principal units of square measure? the Table.

NOTE 1. The hectare, which is a common unit for land measure, is a square whose side is a hundred meters; hence it is equal to 10,000 square meters. It is 2.471 acres.

NOTE 2. Since the scale in square measure is 100 (two dimensions, 10 × 10), there will be two figures for each denomination. Thus, 25 hectares, 7 ares, 17 centares, and 20 square decimeters, would be written 2507.172 ares, ǒr 250717.2 square meters.

Ex. 1. Reduce 15 hectares to sq. meters. Ans. 150,000.

2. Reduce 456000 sq. decimeters to ares. Ans. 45.6.

3. Reduce 78 kilometers to areas.

4. Reduce 9624 ares to hectares. Ans. 96.24.

5. In a field 300 meters long and 78 meters wide, how many ares? Ans. 234.

6. How many hectares in a field 275 meters long and 500 meters wide? Ans. 13.75.

CUBIC MEASURE.

208. The principal unit of cubic measure is the *Cubic Meter* or *Stere*. It is 1.308 cubic yards.

TABLE.

1000 Cub. Centimeters	make	1 Cub. Decimeter, or *Liter*.
1000 Cub. Decimeters	"	1 Cub. Meter, or *Stere* (st).

The tenth part of the Stere is the Decistere, and ten Steres make a Dekastere.

NOTE. Since the scale is a thousand (three dimensions, 10 × 10 × 10), three figures will be required for each denomination.

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207. The common unit for land measure? Equal to how may acres? How many figures required for each denomination? Why?

208. What is the principal unit of cubic measure? Equal to how many cubic yards? Give the Table. How many figures required for each denomination? Why?

Ex. 1. Reduce 12 cubic meters to cubic decimeters. Ans. 12000.

2. Reduce 41,930,000 cubic centimeters to cubic meters or steres. Ans. 41.93^{st}.

3. Reduce 29 steres to decisteres. Ans. 290.

4. Reduce 495 steres to dekasteres.

5. Reduce 5230 decisteres to steres. Ans. 523.

6. In a pile of wood 10 meters long, 1 meter wide and 3 meters high, how many steres? Ans. 30.

7. How many cubic meters in a box 3 meters long, 1.25^{m} wide and 1.2^{m} deep?

DRY AND LIQUID MEASURE.

209. The principal unit, both for Dry and for Liquid Measure, is the Cubic Decimeter, or *Liter*. It is a little larger than a wine quart.

TABLE.

10 Milliliters, or cub. centimeters, make 1 *Centiliter* $(^{cl})$.
10 Centiliters " 1 Deciliter.
10 Deciliters " 1 LITER $(^{l})$, or cub. decimeter.
10 Liters " 1 Dekaliter.
10 Dekaliters " 1 *Hectoliter* $(^{hl})$.
10 Hectoliters " 1 Kiloliter, or cub. meter.

	Cub. Meter.	Hectol.	Dekaliters.	Liters.	Deciliters.	Centiliters.	Milliliters
						1 =	10
					1 =	10 =	100
				1 =	10 =	100 =	1,000
			1 =	10 =	100 =	1,000 =	10,000
		1 =	10 =	100 =	1,000 =	10,000 =	100,000
	1 =	10 =	100 =	1,000 =	10,000 =	100,000 =	1,000,000

---

209. What is the principal unit of Dry and Liquid Measure? How does it compare with a wine quart? Give the Table.

NOTE 1. These measures are usually written as liters and decimal parts; or as hectoliters and decimal parts. Thus, 2 kiloliters, 7 hectoliters, 7 liters, and 5 deciliters are written 27.075^{hl}, or 2707.5^{l}.

NOTE 2. The calculation of the contents of a bin or cistern, is very simple in the metric system. The product of the length, breadth and thickness in decimeters, gives the contents in liters.

Ex. 1. Reduce 24000 milliliters to liters. Ans. 24.

2. Reduce 3560 liters to hectoliters. Ans. 35.6.

3. Reduce .917 liters to centiliters.

4. How many liters in a tank, 4.5^{m} long, 3^{m} wide, and 2.5^{m} deep? $45 \times 30 \times 25 = 33750$. Ans.

5. Reduce 61250 liters to cubic meters. Ans. 61.25.

6. What is the value of a hectoliter of molasses at 45 cts. a liter? Ans. $45.00.

WEIGHT.

210. The principal units of weight are the *Gram* and the *Kilogram*. The Kilogram is the weight of a liter of water, and is a little more than $2\frac{1}{5}$ pounds avoirdupois.

TABLE.

10	Milligrams (^{mg})	make	1	Centigram.
10	Centigrams	"	1	Decigram.
10	Decigrams	"	1	GRAM (^{g}).
10	Grams	"	1	Dekagram.
10	Dekagrams	"	1	Hectogram.
10	Hectograms	"	1	KILOGRAM, or Kilo, (^{kg}).
10	Kilograms	"	1	Myriagram.
10	Myriagrams	"	1	Quintal.
10	Quintals	"	1	*Tonneau* (^{t}).

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209. How are these measures usually written?

210. What are the principal units of weight? What is a kilogram? Equal to how many pounds avoirdupois? Give the Table.

Ton.	Quin.	Myriagr.	Kilogr.	Hectogr.	Dekagr.	Grams.	Decigr.	Centigr.	Milligr.
								1=	10
							1=	10=	100
						1=	10=	100=	1000
					1=	10=	100=	1000=	10,000
				1=	10=	100=	1000=	10,000=	100,000
			1=	10=	100=	1000=	10,000=	100,000=	1,000,000
		1=	10=	100=	1000=	10,000=	100,000=	1,000,000=	10,000,000
	1=	10=	100=	1000=	10,000=	100,000=	1,000,000=	10,000,000=	100,000,000
1=	10=	100=	1000=	10,000=	100,000=	1,000,000=	10,000,000=	100,000,000=	1,000,000,000

NOTE. A cubic centimeter of water weighs a gram, and a cubic meter of water weighs a tonneau. The kilogram is often called kilo, for brevity.

Ex. 1. Reduce 64.73 kilos to grams. Ans. 64730.

2. Reduce 7490 kilos to tonneaus. Ans. 7.49.

3. Reduce 28500 milligrams to grams. Ans. 28.5.

4. What is the weight of 36 liters of water? Ans. 36^{kg}.

5. If $2^{kg.}$ of sugar cost 80 cts. what will be the expense of $16^{kg.}$ at the same rate? Ans. $6.40

6. If one tonneau of coal cost $10, what will 7½ tonneaus cost?

The *names* of the metric weights and measures are formed according to a simple law, as will be seen by inspection of the following scheme:

LENGTHS.	SURFACES.	CAPACITIES.	WEIGHTS.	RATIOS.
Myria - meter.			Myria - gram.	10000
Kilo - meter.		Kilo - liter.	Kilo - gram.	1000
Hecto - meter.	Hect - are.	Hecto - liter.	Hecto - gram.	100
Deka - meter.		Deka - liter.	Deka - gram.	10
METER.	ARE.	LITER.	GRAM.	1
Deci - meter.		Deci - liter.	Deci - gram.	$\frac{1}{10}$
Centi - meter.	Cent - are.	Centi - liter.	Centi - gram.	$\frac{1}{100}$
Milli - meter.		Milli - liter.	Milli - gram.	$\frac{1}{1000}$

211. The following equivalents of the metric measures and weights have been established by Congress for use in all legal proceedings:

MEASURES OF LENGTH.

METRIC DENOMINATIONS AND VALUES.		EQUIVALENTS IN DENOMINATIONS IN USE.
Myriameter . .	10,000 meters . .	6.2137 miles.
Kilometer . . .	1,000 meters . .	0.62137 mile, or 3,280 feet and 10 in.
Hectometer . .	100 meters . .	328 feet and 1 inch.
Dekameter . .	10 meters . .	393.7 inches.
Meter	1 meter . . .	39.37 inches.
Decimeter . . .	$\frac{1}{10}$ of a meter	3.937 inches.
Centimeter . .	$\frac{1}{100}$ of a meter	0.3937 inch.
Millimeter . . .	$\frac{1}{1000}$ of a meter	0.0394 inch.

MEASURES OF SURFACE.

METRIC DENOMINATIONS AND VALUES.		EQUIVALENTS IN DENOMINATIONS IN USE.
Hectare . . .	10,000 sq. meters	2.471 acres.
Are	100 sq. meters	119.6 square yards.
Centare	1 sq. meter	1550 square inches.

MEASURES OF CAPACITY.

METRIC DENOMINATIONS AND VALUES.			EQUIVALENTS IN DENOMINATIONS IN USE.	
Names.	No. of Liters.	Cubic Measure.	Dry Measure.	Liquid or Wine Measure.
Kiloliter or stere	1000	1 cubic meter	1.308 cu. yards	264.17 gallons.
Hectoliter . .	100	$\frac{1}{10}$ of a cu. meter	2 bu. & 3.35 pks.	26.417 gallons.
Dekaliter . . .	10	10 cu. decimeters	9.08 quarts . .	2.6417 gallons.
Liter	1	1 cu. decimeter	0.908 quart . .	1.0567 quarts.
Deciliter . . .	$\frac{1}{10}$	$\frac{1}{10}$ of a cu. decim.	6.1022 cu. inches	0.845 gill.
Centiliter. . .	$\frac{1}{100}$	10 cu. centimeters	0.6102 cubic inch	0.338 fluid ounce.
Milliliter . . .	$\frac{1}{1000}$	1 cu. centimeter	0.061 cubic inch	0.27 fluid drachm.

WEIGHTS.

METRIC DENOMINATIONS AND VALUES.			EQUIVALENTS IN DENOMINATIONS IN USE.
Names.	**No. of Grams.**	**Weight of what quantity of water at maximum density.**	**Avoirdupois Weight.**
Millier or tonneau	1000000	1 cubic meter	2204.6 pounds.
Quintal	100000	1 hectoliter	220.46 pounds.
Myriagram . . .	10000	10 liters	22.046 pounds.
Kilogram or kilo	1000	1 liter	2.2046 pounds.
Hectogram . . .	100	1 deciliter	3.5274 ounces.
Dekagram . . .	10	10 cubic centimeters . .	0.3527 ounce.
Gram	1	1 cubic centimeter . .	15.432 grains.
Decigram . . .	$\frac{1}{10}$	$\frac{1}{10}$ of a cubic centimeter	1.5432 grains.
Centigram . . .	$\frac{1}{100}$	10 cubic millimeters . .	0.1543 grain.
Milligram . . .	$\frac{1}{1000}$	1 cubic millimeter . .	0.0154 grain.

212. To reduce metric weights or measures to those in customary use.

Ex. 1. Reduce 25 kilos to pounds avoirdupois.

Ans. 55.115 lbs.

Since one kilo is 2.2046 pounds, 25 kilos will be 2.2046 × 25 = 55.115 lbs. Hence, we have the

RULE. *Multiply the number of metric units by the corresponding number in the table.*

2. Reduce 30 meters to inches. Ans. 1181.1.
3. Reduce 18 liters to wine quarts. Ans. 19.0206.
4. Reduce 50 hectares to acres. Ans. 123.55.

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212. How are metric weights and measures reduced to those in customary use?

213. To reduce customary weights and measures to those of the metric system.

Ex. 1. Reduce 195 inches to meters. Ans. 4.953.

Since one meter is 39.37 inches, the number of meters in 195 inches is the number of times that 195 contains 39.37, that is, 4.953 + meters. Hence, we have the

Rule. *Divide the number of the customary denomination by the corresponding number in the table.*

2. Reduce 85 gallons to hectoliters. Ans. 3.217.
3. Reduce 28 ounces avoirdupois to grams. Ans. 7.938.
4. Reduce 674 square yards to ares. Ans. 5.635.

Miscellaneous Examples.

1. Add 65^{m}. and 8000^{mm}. Ans. 73^{m}.
2. Add 58.29^{hl}. and 136^{l}. Ans. 5965^{l}.
3. Add 4^{t}. 60^{kg}. and 3620^{g}. Ans. 4063.62^{kg}.
4. From 4^{kg}. take 371^{g}. Ans. 3629^{g}.
5. From 5^{hl}. take 45^{l}. Ans. 4558^{l}.
6. Multiply 24.5^{mm}. by 160. Ans. 3.92^{m}.
7. Multiply 42.35^{g}. by 40. Ans. 1694^{g}.
8. Divide 9^{l}. by 15^{cl}. Ans. 60.
9. Divide 43.46^{km}. by 106. Ans. 410^{m}.
10. Divide 126^{g}. by 42^{g}. Ans. 3.
11. Reduce 68.49^{ha}. to ares. Ans. 6849^{ar}.
12. Reduce 10t. to pounds avoirdupois. Ans. 22046lb.
13. In 100 cords of wood, how many steres? Ans. 362.4^{st}.
14. What cost 15m. of cloth at $3.00 a meter? Ans. $45.00.

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213 How are the customary measures reduced to those of the Metric System?

15. How many kilograms of sugar at 50 cts. a kilogram can be bought for $17.50?

16. How many kilometers from Boston to Albany, the distance being 200 miles? Ans. $321.86^{km.}$

17. In 4 cubic meters of water, how many gallons? Ans. 1056.68.

18. What is the weight in pounds avoirdupois of 2 cubic meters of water? Ans. 4409.2.

19. If a hectoliter of corn costs $2.70, what is the price of a dekaliter? Ans. $27.

20. How many steres will a pile of wood contain that is 20^m long, 2^m wide and 3^m high? Ans. 120^{st}.

21. Bought 500 hectares of land at $75.00 a hectare and sold it at $100.00 a hectare. What was the whole gain, and gain per cent? Ans. $12500, and $33\frac{1}{3}$ per cent.

22. A meter of cloth costs $5.00; how many yds, can be bought for $205.00? Ans. 44.83

23. How many hectoliters in 94 gallons? Ans. 3.557.

24. What is the cost of a quintal of coffee at 60 cents a kilogram? Ans. $30.00.

25. Mount Washington is 6226 feet above the level of the sea; what is its height in meters?

26. The difference in latitude, between New Orleans and Alton Illinois, is 9 degrees, (one tenth of a quadrant, Art. 203,) what is the distance in kilometers between the places? Ans. 1000^{km}.

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